NATURAL RESOURCES DEVELOPMENT COLLEGE

DEPARTMENT OF AGRICULTURAL BUSINESS MANAGEMENT

PRODUCTION ECONOMICS

(ABM 132) MODULE

FOR ALL SECOND YEAR STUDENTS
OPEN AND DISTANCE LEARNING

MODULE CONTENTS

MODULE OVERVIEW

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INTRODUCTION

Welcome to the ABM132 Module on *Production Economics*. In the ABM111 Module you covered basic theories of both microeconomics and macroeconomics. One of the issues covered in ABM111 was on the primary questions of choice. These questions are "what to produce," "How to produce," and "For whom to produce." These questions must be answered both at the individual level by individual firms, and also at aggregate or national level. As such these questions apply at both the micro and macro levels.

Production economics focuses on how individual firms deal with these questions. It looks at how a firm allocates its resources. Production economics therefore deals with technical or mathematical relationships between inputs and output. For any particular method or technology, production economics is concerned with determining the optimum amounts of inputs or output, i.e. the level of inputs or output that maximizes profit or that minimizes costs. This module will therefore walk you through some issues of production economics. The Unit One looks at the principles of production with focus on the classical production function which shows the relationship between a single input and an output. The Unit Two looks at costs and their various classifications. Unit Three looks at another form of the production involving two inputs that combine to produce an output. Unit Four looks at the production possibility frontier, i.e. the production of two variable outputs. The module ends with Unit Five which looks at the issue of risk and uncertainty in agriculture.

In this module, you will enhance your knowledge and understanding of production economics. As you go through the module, you will:

- Appreciate the meaning of production economics.
- Learn the various ways in which individual producers allocate resources as they decide on what to produce.
- Understand the diffierent input-output relationships as well as the criteria for choosing the optimum level of input or output.
- Understand the risks associated to agricultural businesses and how to guard against such risks and uncertainty.

MODULE LEARNING OUTCOMES

Learning outcomes are statements that tell you what knowledge and skills you will have when you have worked successfully through a module.

Knowledge
When you have worked through this module you should be able to:
- Explain the concepts production economics
- Distinguish the various input-output relationships
- Describe how firms determine how much of an input to use or how much to produce
- Recognize the different risks and uncertainties and how to minimize losses arising from such risks and uncertainty

Skills

When you have worked through this module you should be able to:
- Compute the various input and output optima in determining cost minimizing input levels or profit maximizing output levels.
- Analyse the effect price changes on the input and output optima.

ABOUT THIS MODULE

This module is divided into five units:

Unit 1: Principles of production
In this Unit you will be introduced to the factor-product relationship. This is production in which output depends on a single variable input. Unit 1 therefore deals with deciding on how much of an

input to use in order to obtain the highest output, or the highest profit given the prices the input and output.

Unit 2: Costs of Production
In this Unit you will be introduced to the different classifications of costs. You will also look at how information on costs can be used to determine the optimal levels of inputs and output.

Unit 3: Production with Two or More Variable Inputs
Unlike Unit 1 which focuses on output that depends on a single variable input, Unit 3 deals with output that depends on two variable inputs. This Unit presents you with analyses involved in choosing the cheapest (or least cost) combination of two variable inputs.

Unit 4: Production of Two Variable Outputs
This Unit deals with decisions on enterprise combinations, that is the product-product relationship. In this Unit you will learn how to determine combinations of two products that maximise revenue.

Unit 5: Uncertainty and Risks in Agriculture
Production is done over a preriod of time. Most future outcomes may not be known. As such this Unit will introduce you to issues of uncertainty as well as risks faced by firms in agricultural production.

The table below shows you which units cover the different module learning outcomes.

Module Learning Outcomes	Units				
	1	2	3	4	5
Knowledge					
1. Explain the concepts production economics	●	●	●	●	●
2. Distinguish the various input-output relationships	●	●	●	●	
3. Describe how firms determine how much of an input to use or how much to produce	●	●	●	●	
4. Recognize the different risks and uncertainties and how to minimize losses arising from such risks and uncertainty					●
Skills					
1. Compute the various input and output optima in determining cost minimizing input levels or profit maximizing output levels.	●	●	●	●	
2. Analyse the effect price changes on the input and output optima.		●	●	●	

ASSESSMENT

This module is divided into five units. Each unit addresses some of the learning outcomes. You will be asked to complete various tasks so that you can demonstrate your competence in achieving the various learning outcomes.

Assessment Methods

Under this module, you will have two assignments and one continuous assessment test. You will write the first assignment and a test during your residential school following the beginning of a semester. These make up your continous assessment (CA) which contributes 30% of your final mark in the course.

- Each assignment will contribute 7.5% of the final mark (15% for the two assigments)
- The continuous assessment test will contribute 15% of the final mark
- A written examination set by the college at the end of the semester will contribute 70% of the final mark.

LEARNING TIPS

Duration: You will most likely take about 40 hours to work through this module. This includes the time you will spend on the activities and self-help questions.

Activities: This module has several activites under the various units. These activities are meant help you revise the various aspects of the course that you will have covered. This is a way by which you will prepare yourself for the written examination. In addition, the activities offer you an opportunity to apply or practice the various concepts you will come across. You are therefore encourage to perform these activities.

Unit Summary: Each unit ends with a summary of the various issues covered in that unit. The summary reminds you about the key points that you will have covered. Should you feel that the summary has points that you do not remember, consider revisiting sections that cover such points within the unit.

Icons: This module uses a number of icons. Icons are pictures or symbols that serve as indicators or guides that tell you what you should do. Below are the icons you might find in the module:

 Activity Complete the activity	 Note It (Indicates Important points)	 Self-help (Answer the question)	 Summary (A review of the unit)

STUDYING AT A DISTANCE

This being your second semester, you have some experience regarding the demands of distance learning. However, it is important to point out a thing or two regarding time management. Unlike your counterparts in full time programs, you have to make your own time-table. It is therefore important that you apportion your time in such a manner as to afford yourself enough time to go through this module. The module contains suggestions regarding how much time should be spent on a given topic or activity. These are merely guides. You should not only aim at going through the module on the basis of the suggested time, but should also ensure that you meet the expected learning outcomes.

IF YOU NEED HELP

It will be necessary for you to contact us by phone or email for various consultations. As such enquire from the ODL Cordinatior regarding the telephone numbers that you may have to call. For email correspondences, use the address: **info@nrdc.biz**
You can also come to the college whenever you are in Lusaka or if you live in Lusaka.

We hope you will find this learning experience exciting and beneficial.

GOOD LUCK!

1.0 PRINCIPLES OF PRODUCTION

Unit Introduction

In this Unit you will be introduced to the factor-product relationship. This is production in which output depends on a single variable input. The Unit will introduce you to the algebraic production functions as a way of expressing factor-product relationships.

Furthermore the Unit has a number of self-help questions that you will have to undertake. These tasks will enhance your understanding and application of the concepts covered in the Unit.

The Unit ends with a summary of the main points you will have covered.

Unit Learning Outcomes

When you have worked through this Unit, you should be able to:
- Define production and production economics.
- Explain the goals and assumptions of production economics
- Work with algebraic production functions as well as its derivatives
- Determine the three regions or stages of a classical production function

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To begin with we can go through some concepts of economics. In the ABM111 Module, economics was defined as a social science concerned with the allocation of limited resources in order to satisfy unlimited human wants. This definition has key elements namely allocation, scarcity, and choice.

<u>Allocation</u> has to do with putting resources and products to their best use. Generally, <u>price</u> performs the function of allocating resources and output. On the basis of price, goods can either be economic if they have to be paid for or non-economic if they do not command a price.

<u>Scarcity</u> refers to the fact that there are not enough resources or goods to satisfy all desires that people have. As a result of scarcity, individuals and society must deliberately establish goals or choices. This is because any society has many wants which have to compete for the few available resources – as such choices must be made in terms of which wants to satisfy and which ones to give up (forgo or sacrifice).

Economics, therefore, provides tools that help society to decide how to use of its resources based on its goals/choices.

Another important issue is Time. Any production process is influenced by time. Time gives rise to the concepts of consumption and savings. Individuals or society must choose between consumption and saving. Saving implies less consumption now so as to have more later on. Whether you consume now or save is a matter that you should decide upon – what matters is that your choice must be rational, your decision should be such that you maximize satisfaction over time.

PRODUCTION AND PRODUCTION ECONOMICS: Production is the process of combining and coordinating inputs (resources or factors of production) in the creation of a good or service (Colman and Young, 1997). Production economics, also called the theory of the firm, is the study of how firms allocate their scarce resources between alternative uses in order to maximise profit (Hill, 1980). Production economics is regarded as the theory of the firm because it tries to explain the behaviour of firms and their motive of maximising profit.

THE PROCESS OF ECONOMISING

This refers to the ways in which scarce resources can be utilized in order to maximise satisfaction or benefits. It involves three categories of decision concerning inputs and outputs:

a.) Getting more output from a given total amount of inputs: this is attained by allocating money on inputs until the last kwacha spent on each input adds as much to the total output as a kwacha spent anywhere else;
b.) Getting the same amount of total output by using fewer inputs: this is attained by replacing less productive inputs with productive ones until each additional unit of input used yields the same net addition to output; and
c.) Getting more output by increasing output relatively more than inputs: this is attained by using more units of an input as long as each additional unit adds more to the total output to cover the cost of such a unit.

GOALS OF PRODUCTION ECONOMICS

Many issues characterise most economies today. These include availability of many technologies, increased competition, the difficulty of correcting or regulating most mistakes during production, as well as the changing needs, values, and goals of society. Production economics, therefore, seeks to assist decision makers in:

a.) determining the best or optimal use of resources;
b.) determining the consequences that alternative public policies will have on output, profits, and resource use;
c.) improving the management of the farm by understanding the behaviour of a farm firm as a profit maximizing entity; and
d.) determining how individual firms (or aggregate of firms) adjust their output and resource use as a result of changing economic variables.

MAJOR ASSUMPTIONS IN PRODUCTION ECONOMICS

The following are the assumptions will be used as a way of having manageable analyses:

a.) Stability: various economic variables such as technology, government policy, and prices are taken to be stable during whatever period of interest;
b.) Inputs and outputs are assumed to be homogeneous, and perfectly divisible. Being homogeneous implies that all units of an input are of the same quality and therefore equally productive. Similarly all units of output are the same and as such have an equal marketable value. Divisibility implies that units of inputs and output can be used or measured in fractions; (this is true for certain inputs or outputs such as fertilizer and water which are perfectly divisible. It is however not true for other discrete inputs and outputs e.g. a tractor or an ox).
c.) Perfect or absolute certainty: In practice, agricultural production involves many inputs that cannot be controlled such as weather and rainfall, as well as many processes that are not completely understood. These difficulties are avoided by assuming that a producer will know all about inputs required for production as well as the eventual outcome of the production process at the start of the production period;
d.) Level of Technology: A product can be produced in many different ways each of which represents different technologies. It is assumed that producers use the most efficient process available to them i.e. one that results in the most products from a given amount of input;
e.) Length of the time period: Any production process takes place in a given period of time. The assumption in Production economics is that firms are operating in the short run. As such, some inputs or costs are fixed while others are variable. Inputs are fixed if their quantity cannot be reduced or increased <u>during the production period</u>; and
f.) Managers of production processes are motivated by profit in their actions and decisions. This means that the goal of each and every firm is to maximise profit.

TYPES OF PRODUCTION DECISIONS/TYPES OF INPUT-OUTPUT RELATIONSHIPS

Production economics focuses on three types of input-output relationships. These relationships

a.) <u>Factor-product relationship</u>: This is a relationship in which output depends on the amount of a single variable input that is combined with a set of fixed resources. The objective of this relationship is to determine the <u>optimal</u> quantity of the variable input that should be used in combination with the fixed inputs e.g. how much fertilizer to apply per acre. The term "Optimal" implies cost minimization or profit maximization.
b.) <u>Factor-Factor relationship</u>: This is a relationship in which output is dependent on two or more variable inputs combined to a set of fixed resources. The decision to be made in this case is to determine how much of each variable input to use – hence the term "factor-factor." The issue of how these variable inputs substitute each other is also considered. Examples of factor-factor decisions include choosing

different types and sizes of tractors. In choosing how much of each substitute to use, costs as well as the marginal productivity of each input are used to decide.

c.) Product-Product relationship: This is a relationship in which two or more products are dependent on a given set of resources. In this case the decision is on how many enterprises (or product lines) to have and how many resources to allocate to each enterprise. The question asked is not how inputs should be allocated within an enterprise but rather what combinations of enterprises should be produced from a given batch of variable and fixed resources. (Production possibility frontiers are used in product-product analysis.)

1.1 THE CLASSICAL PRODUCTION FUNCTION

Production: the process of creating an economic good or service (referred to as output) from two or more other goods or services (referred to as inputs/resources/factors of production). Inputs can be combined in several different ways. For any given combination of inputs, there is a certain maximum amount of output that can be created – i.e. there is a technical relationship between inputs and output.

A firm's output depends upon the quantities of inputs used in production. This relationship between input and output is represented by what is called a production function. A production function is mathematical relationship describing the way in which the quantity of a particular product depends upon the quantities of particular inputs used. A production function provides information concerning the quantity of output that may be expected when particular inputs are combined in a specified manner, i.e. it shows how much output will be produced by a specific set of resources in a given period of time and state of the arts (or technology).

It is the job of research and experimentation to discover the production functions (which are chemically, physically, and biologically possible). Once these production functions have been discovered, they provide very useful information for making decisions by farmers and other producers. Note that producers do not control the production function (Bishop and Toussaint, 1976).

A production function can be expressed as a schedule (much like demand and supply), or as an algebraic equation such as $Y = f(X_1,X_2,X_3,\ldots,X_n)$ where Y is the physical quantity of output; the symbol f( ) means "is a function of," "results from," or "depends on"; X_1 to X_n are the different inputs used to produce Y.

Inputs can also be specified in terms of whether they are fixed (when their level of use does not change with changes in the levels of output) or variable (when their level of use changes as the level of output changes). With respect to fixed and variable inputs, the equation of the production function becomes: $Y = f(X_1,X_2 \mid X_3,X_4,\ldots,X_n)$, the vertical line separates variable inputs X_1 and X_2 from fixed inputs X_3 to X_n.

FACTOR-PRODUCT RELATIONSHIP

If it is assumed that the quantity of all inputs except one (say, fertilizer, denoted as X_1) is already set or fixed at some level, the relationship between output and the single variable factor can be derived. This factor-product relationship is denoted by the production function $Y = f(X_1 | X_2, X_3, X_4, \ldots, X_n)$ where $X_2, \ldots, X_n$ are the fixed factors; X_1 is the variable factor. This can be seen as a case of a farmer who has just planted a field of maize; fertilizer application is taken as the only factor that he can change in order to affect the yield since all other factors are fixed (the size of the field, quantity of seed already planted, and the labour required).

In this case, as more fertilizer (X_1) is applied, output (Y) increases until a maximum is reached. Further applications of fertiliser will reduce the total quantity produced. This relationship can be expressed as a graph plotting the quantity of output (Y) on the y-axis and the amount of the input (X_1) on the x-axis. The resulting graph is called the total product curve. Total physical product (TPP) refers to the maximum amount that can be produced with any given combination of fixed and variable inputs. Generally, an increase in variable inputs will increase the TPP, but usually not by the same (or direct) proportion – i.e. a 5% increase in a variable input will generally increase TPP but not by 5%, TPP may increase by a less, equal, or greater proportion than 5%.

Example: A farm firm with a purchased variable input and a fixed input consisting of one section (600 acres) of cropland is faced with the following production function:

Variable Input: (X_1)	Fixed Input: 600 acres of Cropland (X_2)	Total Physical Product (TPP)
0	1	0.0
2	1	3.7
4	1	13.9
6	1	28.8
8	1	46.9
10	1	66.7
12	1	86.4
14	1	104.5
16	1	119.5
18	1	129.6
20	1	133.3
22	1	129.1

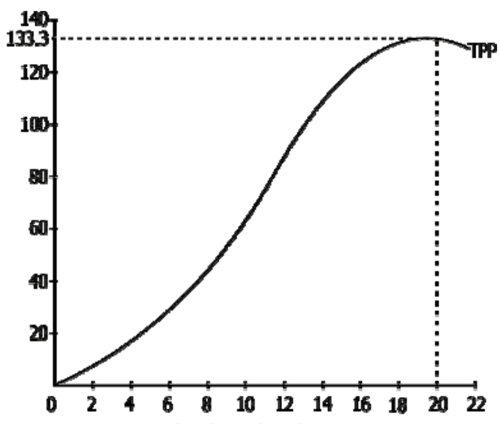

Figure 1.1 The Classical Production Function

The graphical representation of the above input-output schedule is shown in Figure 1.1. The shape of the production function describes the change in output Y as increasing amounts of the variable input X_1 are added to a bundle of fixed factors.

The production function shows that output is zero when the level of the variable input is zero. Output increases at an increasing rate when the initial units if X_1 are added. It continues to increased but at a decreasing rate as more

units of X_1 are added.

The maximum output of 133.3 results from 20 units of X_1. Addition of more units of the variable input eventually results in a decrease in output as is the case at 22 units of X_1.

The continuous production function in Figure 1.1 example above can be expressed in algebraic terms as $Y = X^2 – X^3/30$.

Two measures that can be derived from the production function are the average physical product (APP) and the marginal physical product (MPP).

1. Average Physical Product (APP): This represents the contribution made by one unit of input to the total output on the basis that all units of the input contribute equally. For any level of output, APP is measured by dividing the total output produced over a given period by the number of units of the input used to produce that level of output. Thus APP = Y/X. Given the example that $Y = X^2 – X^3/30$, $APP = Y/X = X – X^2/30$

2. Marginal Physical Product (MPP): The MPP of an input is the contribution made to total output by the use of one more unit of that input when the quantity of all other inputs is unchanged. It is a measure of how much one unit of an input contributes to the total output produced. MPP can be measured in two ways:
   i.) The average method: this is used when working with tabular data. In this case MPP is computed by dividing the change in output by the causal amount of the input: $MPP = \Delta Y/\Delta X$. For example, MPP in moving from 10 to 12 units of X_1 (in our above example) is found as $\Delta Y/\Delta X = (86.4 – 66.7)/(12 – 10) = 9.85$; meaning that when 10 units of X_1 are already being used, use of one more unit will add 9.85 more units to total output. This method is called "average" because its measures MPP over a range of values i.e. there are many values of MPP between 10 and 12 units of X1.
   ii.) The exact method: This uses calculus and requires algebraic production functions. For any equation of Y (or TPP), the equation for MPP is obtained as the first derivative. For $Y = f(X)$, $MPP = \delta Y/\delta X$. Using the above example of $Y = X^2 – X^3/30$, $MPP = 2X – X^2/10$. With this method, the MPP for any level (and not a range of values) of X can be obtained - hence the name "Exact Method."

   MPP (being $\Delta Y/\Delta X$) measures the slope of the TPP curve. The shape of the MPP curve therefore describes the changing nature of the TPP curve. TPP increases at an increasing rate when MPP is increasing; increases at a decreasing rate when MPP is falling; reaches a maximum when MPP is zero, and declines when MPP is negative.

THE LAW OF DIMINISHING RETURNS (OR OF VARIABLE PROPORTIONS)

This describes the relationship between output and a variable input when other inputs are fixed. It sates that "if increasing amounts of one input are added to a production process while other inputs are held constant, the amount of output added by each successive unit of input eventually decreases." In other words, as more units of a variable input are used, the contribution to TPP of each additional unit eventually decreases. The law implies that the MPP eventually declines. The law of variable proportions is based on two assumptions:

ii.) the technology used remains unchanged; and
iii.) some inputs are fixed – the law does not hold or apply when all inputs are varied.

On the basis of marginal returns (i.e. the productivity of each additional unit of the input), four types scenarios can be identified (refer to Figure 1.2 on the next page):

i.) <u>Increasing marginal returns</u>: these occur from O to A. In this portion, each unit of the input used adds more to the total output than the unit before it. As such TPP increases at an increasing rate.
ii.) <u>Constant marginal returns</u>: these occur when each unit of an input used adds an equal amount to TPP as all the other units. This gives a straight line TPP graph – and might arise where there is an unused surplus of the fixed resources. In this case MPP is a horizontal line.
iii.) <u>Decreasing marginal returns</u>: Occurs from A to C. In this case each additional unit of the variable input adds less and less to total output than the unit before it i.e. additional units become less productive successively until the last unit adds nothing to the output. In out example, the 20^{th} unit of X_1 adds zero to TPP, hence MPP=0 at X_1=20. The Law of diminishing returns starts operating in the marginal sense at A i.e. <u>diminishing marginal returns</u> set in at A. This point is called the inflection point, and is where MPP reaches its maximum.
iv.) <u>Negative marginal Returns</u>: This occurs from point C and beyond in which case adding more units of the variable input reduces the total output.

1.2 STAGES OF PRODUCTION

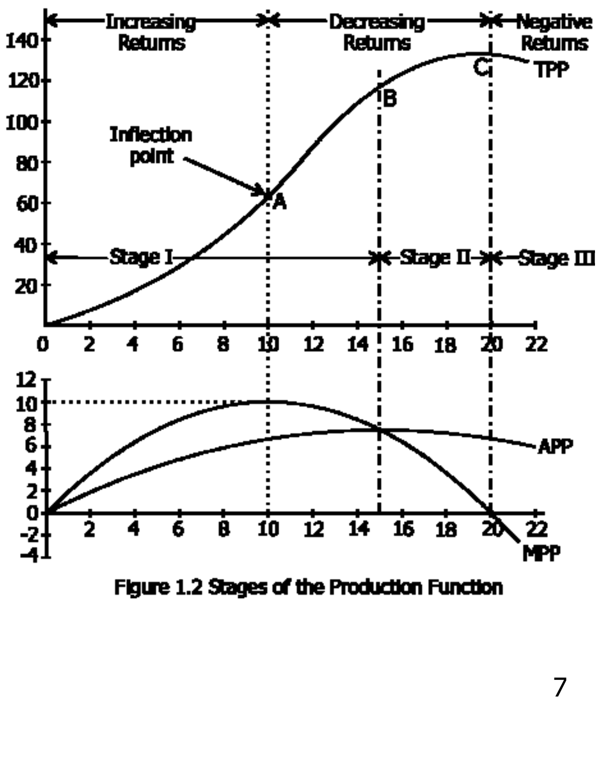

Figure 1.2 Stages of the Production Function

A classical production function (classical because it exhibits the whole range of input productivity from increasing to negative returns) can be divided into three regions or stages on the basis of efficient resource used.

STAGE I: This starts from 0 units of input up to the point where APP is at its maximum – point B on the diagram above. In this stage MPP is above APP; it reaches its maximum at the inflection point (where the marginal productivity of the variable input is at its highest) and starts falling to a point where it meets APP marking the end of Stage I. Thus at the end of Stage I, MPP=APP, and APP is at its maximum (meaning that the average productivity of the variable input is at its highest).

In algebraic terms, Stage I ends when the slope of APP=0 (i.e. when APP is at its maximum). Using our example of $Y=X^2-X^3/30$, $APP=X-X^2/30$, thus the slope of APP is $\delta APP/\delta X=1-X/15=0$ when APP is at its maximum. Thus at X=15, APP=7.5 and is at its maximum.

At X=15, MPP given as $2X-0.1X^2$ is also equal to $2(15)-(15^2)/10=7.5$ proving that APP=MPP, when APP is at its maximum.

STAGE II: This starts where APP is at its maximum, and equal to MPP. In this stage APP is above MPP and the two are falling. Stage II ends when MPP=0. Since MPP measures the slope of the TPP, TPP will be at its maximum when MPP=0.

Using our example, $MPP=2X-X^2/10=0$ when TPP is highest; From $2X-0.1X^2=0$, $X(2-0.1X)=0$; thus X=0 or X=2/0.1=20. Since TPP cannot be at its highest at X=0, then X=20 is the acceptable level of input.

STAGE III: Starts where MPP=0. In this stage MPP is negative and this occurs when excessive quantities of the variable input are combined with the fixed input so much that the total output begins to decrease.

<u>SELF HELP QUESTION 1</u> (15 minutes)
Experimental data shows that maize yield (Y) in 50Kg bags responds to nitrogen (X) in kilograms in the following way: $Y = 37X + 0.8X^2 - 0.001X^3$,

- Determine the kilograms of nitrogen that will indicate the boundaries of the three stages of production;
- Determine the stage of production when X = 15Kg and prove that you are indeed in the correct stage using the concept of elasticity of production. (Read next page for elasticity of production).

CHOOSING THE OPTIMAL LEVEL OF PRODUCTION

When the technical relationship between input and output is known, some recommendations can be made concerning the use of an input even when prices are not specified:

1. If the product has any value at all, input use once began must be continued until Stage II is reached. This is because the physical efficiency of the variable input (as measured by APP) increases throughout Stage I. In Stage I, MPP>APP implying that with each additional unit of input more is being added <u>on average</u> to TPP than was before. Therefore it will pay to produce at least up to the end of Stage I.
2. Even if the input is free, it should not be used in Stage III because instead of increasing output, further use of the input actually reduces the total output. This is indicated by the fact that MPP is negative in Stage III.

3. Stage II is therefore the only rational stage of production and as such defines the area of economic relevance. The amount of variable input should be used must be somewhere in Stage II but the exact amount can only be determined when input and output prices are known.

ELASTICITY OF PRODUCTION & THE POINT OF DIMINISHING RETURNS

The law of diminishing returns is not clear about the level of input or output at which returns begin to diminish. For example MPP begins to decrease at input X=10; APP begins to decrease at X=15, while TPP begins to decrease at X=20. Clearly the point of diminishing returns depends on which of these three measures is being considered. To avoid this confusion, the law of diminishing returns is applied directly to the MPP – and is thus called the law of diminishing marginal returns which states that as more units of a variable input are added, the contribution to total output made by each additional unit of the input eventually decreases. However, this can still be confusing because the point of diminishing marginal returns at X=10 (in our example) differs from the boundary of Stages I and II where X=15. This is resolved by using the elasticity of production – a concept which measures the degree of responsiveness of output to changes in input level. Elasticity of production, E_p = (% change in output)÷(% change in input)

$$E_p = \frac{\Delta Y}{Y} \div \frac{\Delta X}{X} = \frac{\Delta Y}{Y} * \frac{X}{\Delta X} = \frac{\Delta Y}{\Delta X} * \frac{X}{Y} = \frac{MPP}{APP}$$

The boundaries of the production function can be defined on the basis of elasticity of production. In Stage I $E_p>1$ because MPP>APP. The boundary between Stages I and II is when $E_p=1$ (because at this point MPP=APP). Throughout Stage II, E_p is less than one but greater than zero (because MPP<APP). At the boundary of Stages II and III, $E_p=0$ because MPP=0. In Stage III, MPP is negative and as such E_p is also negative.

With respect to the elasticity of production, diminishing returns of the variable resource are said to set in when $E_p=1$ (when MPP=APP) and this is the end of Stage I. It gives the minimum amount of the variable input that should be used.

UNIT SUMMARY

In the factor-product relationship, the production function Y=f(X) is a relationship in which output Y depends on a single variable input X. Other inputs are regarded as fixed.

APP is obtained as Y/X where Y is the equation of the production function; it measures the contribution of each unit of the input to the output *on the basis that each input contributes equally*. The input is efficient when each unit added increases the APP. MPP is obtained as $\Delta Y/\Delta X$ or more exactly as $\delta Y/\delta X$; it measures the actual contribution to TPP made by each unit of the input used.

APP and MPP can be used to identify the three stages of production. They are also used to compute the elasticity of production E_P. The goal in analysing the factor-product relationship is to determine the optimal amount of the variable input to use. This level lies in stage II, but the actual quantity can only be known when input and output prices are known. This is what you will look at in the next Unit.

2.0 COSTS OF PRODUCTION

Unit Introduction

Having successfully gone through Unit 1, you are now going to be introduced to the costs of production. During your study of Basic Economic Theory, you learned about the four factors of production. These factor or inputs have to be paid for, that is they have different returns. These returns are costs to the firm. Firms also must sell their output in order to generate revenue. It is this revenue that firms use to pay for the costs of production. In this Unit you will look at how information on costs or prices can be used to determine the optimal levels of inputs and output.

Unit Learning Outcomes

When you have worked through this Unit, you should be able to:
- Identify and describe the different types of costs.
- Distinguish between economies and diseconomies of scale
- Derive avarage and marginal cost equations from the total cost equations
- Compute the profit maximizing level of a variable input or output

In the previous Unit, you covered the fact that the optimum level of the variable input lies somewhere in stage II of the production function. The introduction of costs or prices will now enable you to determine the exact quantity of an input that must be used in order to maximize profit. Remember profit maximization is assumed to be the goal of every firm.

The issue of costs in economics is based on the fact that resources are scarce and have alternative uses. Generally, costs are moneys spent or expenses incurred in organizing and carrying out the production process. Using resources to produce a certain product means that other alternative products must be forgone – production therefore involves <u>opportunity costs</u>. On the basis of the opportunity cost, economic costs are payments that a firm must make in order to attract and keep resources away from their alternative uses. These payments may be explicit or implicit. <u>Explicit costs</u> are those payments that involve an actual money exchange e.g. when payments are made for such things as hired labour, rented land, or purchases of farm supplies. The costs of resources that are owned and therefore that may not require to be paid for are <u>implicit costs</u>. Such resources usually include family labour as well as one's own time. Even though no money payments are made, the cost of such resources is the money they could have earned in the best alternative employments.

2.1 CONCEPT OF COSTS: SHORT AND LONG RUN COSTS

One other assumption that was stated in Unit 1 is that firms operate in the short run. In this period of time, each firm's overall or total costs (TC) include fixed costs (FC) and variable costs (VC). Remember that the short run is a period of time too brief to change the capacity/size of a firm but long enough to

change the levels at which the fixed resources are used. The short run might be a single crop season, or several years to permit the life cycle of the product of interest (e.g. 3-5 years for beef cattle).

Fixed Costs: These do not change in magnitude as the amount of output changes and are incurred even when production is not undertaken. Thus FCs are independent of output. In farming cash FCs include land taxes, principle and interest on land payments, as well as insurance premiums. Non-cash FCs include depreciation, as well as charges for family labour and management. Total fixed cost (TFC) is the same for all levels of output. Graphically, TFC is a straight horizontal line parallel to the input axis.

Variable Costs: These are costs which change in total with changes in output. VCs start at zero when output is zero, increasing as output increases up to the end of Stage II of the production function, and continuing to increase if production continues into Stage III. Total variable cost (TVC) is computed by multiplying the amount of the variable input used by the price per unit of the input: if X is the amount of input and P_x is the unit cost of the input, the TVC=P_xX.

Total Cost (TC): This is the sum of fixed and variable costs at each level of output: TC = TFC + TVC or P_xX + TFC. Graphically, the shape of the TC curve is the same as that of the TVC except it starts at level of TFC.

In the long run all costs are variable – as such there are no fixed costs in the long run. The long run is a period of time which is extensive enough to permit firms to change all the resources employed including their production capacity. It is a period of time in which existing firms can go out of business while new ones can enter the market.

2.2 AVERAGE COSTS AND MARGINAL COSTS OF PRODUCTION

Averages are computed by dividing each of the costs by the number of units produced at a particular level of output.

Average Fixed Cost (AFC): This is obtained by dividing TFC by the corresponding level of output: AFC = TFC/Y. AFC declines as output increases. Fixed costs are sometimes called *Overhead Costs* – and economists sometimes use the phrase "Spreading the overheads" to refer to declining AFC when output increases.

Average Variable Cost (AVC): This is computed by dividing TVC by the corresponding amount of output (Y): AVC = TVC/Y = P_xX/Y; note that X/Y = 1/APP, therefore AVC = P_x/APP. The curve of AVC is U shaped indicating that AVC initially declines, reaches a minimum and then increases. As can be noted in the formula AVC=P_X/APP, APP and AVC have an inverse relationship i.e. when APP is increasing AVC is decreasing; when APP is at its maximum, AVC is at its lowest; when APP is decreasing, AVC is increasing.

Average Total Cost (ATC): This is computed as TC/Y or as AFC+AVC at each level of output. ATC (Average Cost or AC) is often referred to as the unit cost of production i.e. the cost of producing one unit of output. ATC decreases as output increases from zero, reaches its minimum and thereafter

increases. The initial decrease in ATC is caused by the spreading of overheads as well as the increasing efficiency with which the variable input is used.

Marginal Cost (MC): This is the extra or additional cost of producing one more unit of output. It is the change in total cost per unit increase in output. $MC=\Delta TC/\Delta Y$. Since TC consists of TFC and TVC, and that TFC does not change, the change in TC is due to the change in TVC – as such, MC can also be computed as $\Delta TVC/\Delta Y$. Graphically, MC is the slope of the TC curve and also of the TVC curve.

Since $MC= \Delta TVC/\Delta Y$, $\Delta TVC=P_x\Delta X$; Thus $MC= P_x\Delta X/\Delta Y$. Note that $\Delta X/\Delta Y=1/MPP$, therefore $MC=P_x/MPP$. MC is as such inversely related to MPP: when MPP is increasing, MC is decreasing; when MPP is at its maximum, MC is at its minimum; when MPP is falling, MC is increasing.

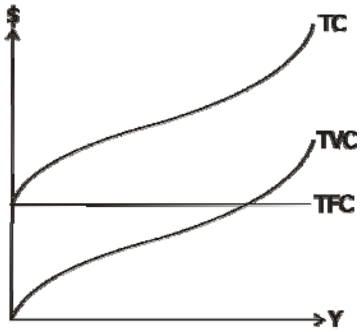

Figure 2.1 Total Costs

NOTE: All costs are usually computed as functions of output, i.e. costs depend on the level of output. This means that in any cost function or equation, cost is the dependent variable while output (Y) is the independent variable: cost = f(Y). Thus graphs of cost curves are plotted using the money values of the Y-axis and the units of output on the X-axis as shown in Figure 2.1.

Points to note:

1. Marginal cost is important because it indicates how much the firm must pay (i.e. how much must be added to the total cost) in order to get ***one more unit*** of output – average costs cannot provide this information. A good manager must try to develop a way of measuring MC at each level of output. MC reflects and is a result of increasing and diminishing returns as can be seen by comparing it with the marginal product curve: increasing returns (as shown by the rising MPP) will be reflected in a decreasing marginal cost while diminishing returns (as shown by the falling MPP) will be reflected in an increasing marginal cost. This can be seen in Figure 2.2.
2. The relationship of MC to AVC and ATC is that MC cuts both AVC and ATC curves from below when each curve is at its minimum. When AVC and ATC are falling, MC will be under them – this is because less is being added to the total cost for each successive unit of output produced. As soon as MC curve cuts the AVC and ATC curves from below, each of these curves will begin to rise.

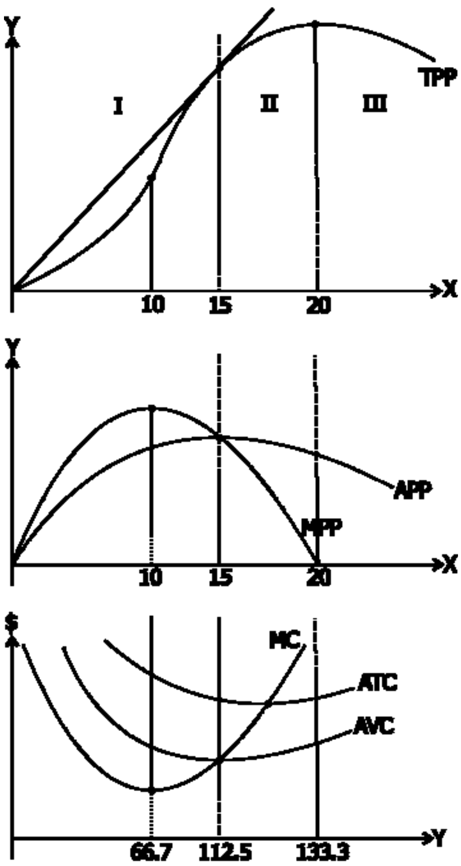

Figure 2.2 Production and Cost Functions

COST RELATIONSHIPS IN THE THREE STAGES OF PRODUCTION

Figure 2.2 (on page 12) shows that AVC is the inverse of APP, just as MC is the inverse of MPP. It is therefore possible to determine some stages of production using the cost functions. The boundary between Stages I and II is where marginal cost is equal to average variable cost i.e. MC=AVC. As such Stage II begins when MC=AVC and ends when output is at its maximum. In Figure 2.2 these limits are at 112.5 and 133.3 units of output. On the boundary of Stages II and III, MPP=0; the MC curve becomes vertical as it approaches the end of Stage II and is therefore undefined (i.e. has no value) at this level.

Given that the total cost equation is $TC=100+6Y-0.4Y^2+0.02Y^3$,
Total Fixed Cost, TFC=100, it is the constant term in the TC equation.
Thus Total Variable Cost, $TVC=6Y-0.4Y^2+0.02Y^3$, it is the rest of the equation excluding the 100
$AVC = TVC/Y = (6Y-0.4Y^2+0.02Y^3)/Y = 6-0.4Y+0.02Y^2$,
$MC=\Delta TC/\Delta Y=\delta TC/\delta Y=6-0.8Y+0.06Y^2$
MC is also equal to $\delta TVC/\delta Y$.

Stage II starts where AVC is at its minimum, i.e. when they slope of $6-0.4Y+0.02Y^2$ is zero. The slope of AVC is given as: $\delta AVC/\delta Y= -0.4+0.04Y$.
Hence AVC will be at its minimum when $-0.4+0.04Y=0$, i.e. $Y=0.4/0.04=10$ giving the value of AVC as $\$6-0.4(10)+0.02(10^2)=\$4$. Since MC=AVC at the beginning of Stage II, replacing Y=10 in the MC equation should also give the answer of MC=\$4 (as follows: $6-0.8(10)+0.06(10^2)=\$4$).

2.3 PROFIT MAXIMIZATION FOR AN ENTERPRISE (FACTOR-PRODUCT RELATIONSHIP)

Now that we have information on the cost of production, we will use this knowledge together with the information on the price of the output (P_Y) to determine the profit maximising level of both the input and the output. This is about finding the level of output that should be produced, or the amount of a variable input that should be used in order to obtain the highest profit. Such a level of a variable input that maximizes short run profits is referred to as the OPTIMUM amount.

2.3.1 DETERMINING THE OPTIMUM AMOUNT OF INPUT

Using Total Value Product (TVP)

In this and other analyses that will follow, the assumption is that a firm operates in a competitive environment in which there are many sellers. As such no single firm can influence the price of the industry (the industry being the total of all firms whose products satisfy the same need). Every single firm has no influence because its output is just a small fraction of the total output for the industry. Hence each firm takes the price of the output (P_Y) as being constant.

TVP is the total money value of a product i.e. the total revenue that a firm earns for selling any given amount of its output. $TVP=P_YY$ with P_Y being the unit price of the output, and Y being the amount of output at any level of the variable in X.

Profit, $\Pi = TVP - TC$

$= TVP - TVC - TFC$

$\Pi = P_YY - P_XX - TFC$

The optimum amount of the variable input is that which results in the largest profits. All the methods of determining the optimum amount of input can be derived from the study of TVP and TC. Consider the following example. This example is based on our earlier example of the production function $Y=X^2-X^3/30$. This time we have information on the unit price of the variable input P_X as \$100, and price of output P_Y as \$30 per unit.

Input X	Output Y	Total Costs (P_X=\$100)	Total Value Product (P_Y=\$30)	Profit TVP-TC
0	0.0	1000	0	-1000
2	3.7	1200	112	-1088
4	13.9	1400	416	-984
6	28.8	1600	864	-736
8	46.9	1800	1408	-392
10	66.7	2000	2000	0
12	86.4	2200	2592	392
14	104.5	2400	3136	736
16	119.5	2600	3584	984
18	129.6	2800	3888	1088
20	133.3	3000	4000	1000
22	129.1	3200	3872	672

From the table above, it can be observed that the highest profit of \$1088 can be obtained when 18 units of the variable input are used.

<u>Using the Marginal Criterion</u>

This method uses the slopes of TVP and TC curves when these curves are plotted as functions of the variable input. The profit equation given above is $\Pi = P_YY - P_XX - TFC$. When profit is at its highest, the slope of the profit equation is zero. Note that this profit equation has two different independent variables X and Y. Like any other equation, we can tell that profit will be at its maximum when the slope of the profit equation is zero, i.e. when the first derivative of the profit equation is zero. It is however difficult to completely solve this equation because of its two different independent variables X and Y. In order to overcome this problem and have an equation that has only one independent variable, we will replace Y by its equation; remember that Y is the output whose equation or the production function has X as the independent variable. As such given that Y=f(X), the profit equation becomes: $\Pi = P_Yf(x) - P_XX - TFC$

F(X) is the equation of Y, and so we have replaced Y by its equation such that the whole equation now only has X as the independent variable.

This slope is given as the first derivative of the profit equation: $\delta\Pi/\delta X=P_Y(\delta Y/\delta X) - P_X$.

TFC is a constant and as such its first derivative is zero. Note that $\delta Y/\delta X$ = MPP, as such the slope of the profit equation can be rewritten as $\delta\Pi/\delta X = P_YMPP - P_X$.

When profit is at its highest, $P_YMPP - P_X = 0$. Reorganising this equation gives $P_YMPP = P_X$ as the profit maximizing condition.

The term P_YMPP measures the slope of TVP. (Recall that TVP = P_YY or $P_Yf(x)$, the slope of TVP is given as $\delta TVP/\delta X = P_Y(\delta Y/\delta X) = P_YMPP$). P_YMPP is called the <u>value of the marginal product </u>(VMP).

Total cost, TC = TVC + TFC = P_xX + TFC
The slope of TC, $\delta TC/\delta X = P_x$.

P_YMPP is the slope of TVP while P_X is the slope of TC. Therefore from our profit maximising condition of $P_YMPP = P_X$, it can seen that profit is maximized when the slope of TVP (as given by P_YMPP) is equal to the slope of TC curve (as given by P_X).

<u>Example</u>: Given a production function $Y= X^2 – X^3/30$, and the unit price of the input and output as \$100 and \$30 respectively, determine the profit maximizing level of the variable input.

<u>Solution</u>: At profit maximization, the slope of the profit equation is zero:
$\delta \Pi/\delta X = P_Y(\delta Y/\delta X) – P_X = 0$; thus $P_YMPP = P_X$
From the given production function, $MPP = 2X – X^2/10$, hence $P_YMPP = P_X$ becomes $P_Y(2X – X^2/10) = P_X$
Replacing P_Y and P_X by their values gives:
$\$30(2X – X^2/10) = \$100$; dividing both side by \$1 gives:
$60X – 3X^2 = 100$; rearranging the equation gives the following quadratic equation:
$3X^2 – 60X + 100 = 0$
Using the quadratic formula $(–b \pm \sqrt{(b^2 – 4ac)})/2a$; with a=3, b= -60, and c=100
<u>X = 1.84 or X = 18.16</u>

MPP at X=1.84 and X=18.16 is 3.34. APP given by $X – X^2/30$; at X=1.84 and 18.16 is 1.73 and 7.17 respectively. Since MPP>APP at X=1.84, this indicates Stage I which is an irrational area to produce in. At X=18.16, 0<MPP<APP indicating that production is in Stage II which is the ration stage in which to produce. Therefore, X=18.16 is the profit maximizing level if the variable input.
We can determine how much to produce by simply replacing X = 18.16 in the production function $Y=X^2–X^3/30$.

2.3.2 DETERMINING THE OPTIMUM AMOUNT OF OUTPUT

In deciding the optimal level of output, total revenue (TR) and total cost (TC) are used. TR is the same as total value of product (TVP) – it is the total amount of money that a firm receives for selling all its units of output, i.e. TR = P_YY.

Profit, Π = TR – TC
= $P_YY – P_XX$ – TFC
This profit equation is expressed in terms of both Y and X i.e. the equation has both Y and X. Since our interest is to find the profit maximising level of output, we should express the profit equation

only in terms of Y, i.e. X must be expressed in terms of Y. You already know that output (Y) is a function of the variable input (X), thus Y can be expressed in terms of X as Y = f(X). X can therefore be expressed in terms of Y by using the inverse equation of Y, i.e. $X = f^{-1}(Y)$. The profit equation becomes:

$\Pi = P_Y Y - f^{-1}(Y) - TFC$

Profit is at its highest when the slope of the profit equation is equal to zero:

$\delta\Pi/\delta Y = P_Y - P_X(\delta Y/\delta X)^{-1} = 0$

$\delta\Pi/\delta Y = P_Y - P_X(MPP)^{-1} = 0$

$P_Y - P_X/MPP = 0$

On page 12, we showed that P_X/MPP = Marginal Cost. So replacing PX/MPP by MC gives our profit maximizing condition as: $P_Y - MC = 0$, or $P_Y = MC$

SELF HELP QUESTION 2 (15 minutes)

Consider the following production function: $Y = 70 + 2X - 0.005X^2$

- Calculate the elasticity of production when X = 40 and interpret your result;
- Find the levels of X that maximizes net returns (profits) when P_X = \$10 and P_Y = \$100.

PRODUCTION COSTS IN THE LONG RUN

Even though we assumed that all firms operate in the short run, we will in this section look at the issue of costs in the long run. The long run is a space of time long enough to allow the firm to change the levels of all inputs. All costs are variable in the long run i.e. AVC=ATC. No resource is fixed so firms can change their plant (factory) sizes or capacity, and the industry can also increase or decrease in size as new firms enter and others leave. Since there are no fixed costs in the long run, the law of diminishing returns does not apply. The behaviour of cost will be mainly influenced by economies and diseconomies of scale. This means that in the long run the average variable cost or the average total cost will still decline and eventually rise. However this decline and rise will NOT be as a result the law of diminishing returns.

There are economies of scale (or increasing returns to scale) when long-run average costs decrease as output increases. Scale refers to the size of the firm as measured by its output. This concept simply indicates that there are economic benefits or "economies" that a firm can enjoy by being big or producing higher levels of output. Reasons for economies of scale include:

a.) Indivisible resources: some inputs cannot be acquired in fractions e.g. labour, tractor, or telephone. At lower levels of output, the cost of these resources compared against the output (i.e. the average cost) will be very high. But as more and more output is produced, these costs are spread over a lot of output and as such the average cost reduces.

b.) Efficiency in the use of labour: This is achieved in two ways. First is specialization (also called the Division of labour): this implies that each worker can concentrate on a single task. In this way the worker will master this task and become efficient in his/her role. Second is when technology

improves: in this case, a worker who was operating a 10hp tractor will now operate a 100hp model and so the cost of his labour can be spread over a greater volume of output.

c.) Economic use of by-products: large scale production can result in high quantities of by-products which may also become of economic value. A butchery that slaughters hundred of cattle in a week may be better placed to economically operate a leather processing factory that one that handles fewer animals.

d.) Increase in the volume of discounts on prices: this is possible because suppliers often reduce prices for customers that buy in bulk.

e.) Growth of supporting services/facilities: As a firm becomes bigger, the local government may provide services (good roads) and other firms may move closer thereby reducing the larger firm's operational costs.

Economies of scale lower the average cost as more and more output is produce. However, a point is reached when an increase in the firm's size begins to cause an increase in the average cost. When this happens, the firm faces diseconomies of scale. There are diseconomies of scale (or decreasing returns to scale) when the long-run average costs increase as output rises. Diseconomies of scale are economic disadvantages that a firm suffers from as it becomes too big. The main reason for this is that management becomes more difficult as the firm becomes larger. Geographical factors might also contribute to diseconomies of scale – for example, the first factory of a firm will be located in the best site (e.g. near input and product markets); but as the firm grows, its second factory be in a less advantageous location and this could increase the firms costs.

UNIT SUMMARY

Costs are moneys spent or expenses incurred in organizing and carrying out the production process. In general, costs depend on how much is produced, and as such costs are a function of output. This means that output (Y) is the independent variable in cost equations. Total costs are made up of fixed costs and variable costs. Average cost is found by diving any cost equation by output (Y).

AVC = TVC/Y as has also be shown to be equal to P_X/APP. Marginal Cost, MC = ΔTC/ΔY or ΔTVC/ΔY. MC has also been shown to equal to P_X/MPP. AVC and MC can be used to identify the boundary between stages I and II; this occurs when AVC is at its lowest <u>and</u> equal to MC.

Given a production function and the unit price of the variable input (P_X) as well as of the product (P_Y), the profit maximizing level of the input is determined when the value of the marginal product (VMP which is P_YMPP) is equal to the unit price of the input (P_X): P_YMPP = P_X

Given the total cost (TC) or total variable cost (TVC) equations as well as the unit price of the output (P_Y), the profit maximizing level of output is determined when the marginal cost (MC) is equal to the unit price of the output (P_Y): MC = P_Y.

3.0 PRODUCTION WITH TWO OR MORE VARIABLE INPUTS

Unit Introduction

In this Unit you are going to learn about the factor-factor relationships. You will deal with making decisions on choosing a combination of two inputs that maximises profit. Your knowledge on the production functions as well as cost will be applied in this Unit.

Unit Learning Outcomes

When you have worked through this Unit, you should be able to:
- Distinguish between isoquants and isocostlines.
- Explain the concept of input substitution
- Explain the concepts of ridgelines and the expansion path
- Algebraically derive the expansion path equation
- Compute the least cost combinations or profit maximization with two variable inputs

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Having successfully looked at the factor-product relationship, you are now going to learn about the factor-factor relationship. This deals with the relationship between one output and two or more variable inputs. In this case, a given level of output may be produced in more than one way. This is especially true for agriculture where a product can be produced using different combinations of inputs. Analysis of the factor-factor relationship is important when having to find the "right combination" of the inputs. Since a given level of output can be produced in many ways, there will be many combinations of inputs each capable of generating the same quantity of output. As such one input can replace the other in order to produce a given level of output, and it is for this reason that production with two or more variable inputs is also referred to as a "Factor-factor Relationship."

The same assumptions used in input-output relationships apply even in factor-factor relationships: The firm operates in a purely competitive market implying that prices are determined by market forces and not by any individual firm. It is also assumed that at least one input must be fixed (so that the law of diminishing returns can operate); the production function will now take the general form of:

$Y = f(X_1, X_2 \mid X_3, X_4, \ldots, X_n)$

Y being the amount of output, X_1 and X_2 being the variable inputs that must be combined with some fixed inputs (X_3 to X_n).

Table 3.1 below shows the different quantities of maize that can be produced by using different combinations of Urea (U) and manure (M). In this table, U increases vertically from 0 to 10 units, M increases horizontally from 0 to 10 units. When we use 0 of M and 10 units of U, the output is 80 units. Notice that each particular level of output is produced by a particular combination of these two inputs. The levels of output are indicated by all the shaded cells.

U	10	80	93	104	113	120	125	128	129	128	125	120
	9	81	94	105	114	121	126	129	130	129	126	121
	8	80	93	104	113	120	125	128	129	128	125	120
	7	77	90	101	110	117	122	125	126	125	122	117
	6	72	85	96	105	112	117	120	121	120	117	112
	5	65	78	89	98	105	110	113	114	113	110	105
	4	56	69	80	89	96	101	104	105	104	101	96
	3	45	58	69	78	85	90	93	94	93	90	85
	2	32	45	56	65	72	77	80	81	80	77	72
	1	17	30	41	50	57	62	65	66	65	62	57
	0	0	13	24	33	40	45	48	49	48	45	40
		0	1	2	3	4	5	6	7	8	9	10
		M										

Table 3.1 Output level for different combinations of urea (U) and manure (M)

These outputs are based on the production function $Y = 18U - U^2 + 14M - M^2$.
Y represents the total physical product (TPP).

Because the variable inputs U and M are combined with some fixed resources, the law of diminishing returns operates. As such, TPP increases at an increasing rate at lower levels of the variable inputs. It reaches its maximum when the MPP <u>for each input</u> is zero (remember that MPP is the first derivative of the production function). The production function has two different independent variables U and M, and we cannot express U in terms of M in order to only have one of them in the equation. This means that we will have two MPPs: one for each independent variable. The MPP for input U is computed by obtaining a derivative of the Y equation in terms of U: $\delta Y/\delta U$
$MPP_U = \delta Y/\delta U = 18 - 2U$, when $MPP_U = 0$, $U = 9$.

The MPP for input M is similarly computed as the derivative of Y with respect to M, i.e. $\delta Y/\delta M$
$MPP_M = \delta Y/\delta M = 14 - 2M$, when $MPP_M = 0$, $M = 7$

This means that output is maximized when 9 units of U are combined with 7 units of M. This maximum output is computed by replacing U = 9 and M = 7 in the production function resulting in Y being 130.

3.1. ISOQUANTS (ISOPRODUCT, OR EQUAL PRODUCT CURVE)

With reference to Table 3.1, you will notice that with the exception of the minimum output of zero and the maximum of 130, all other levels of output can be produced using several combinations of urea (U) and manure (M). For example, the output level of 105 can be produced using the five different combinations (U=9, M=2), (U=6, M=3), (U=5, M=4), (U=4, M=7), and (U=5, M=10) as shown by the darkly shaded cells in Table 3.1.

A curve representing all combinations of variable inputs that produce a given level of output is called an **isoquant**. By plotting the five combinations (U=9, M=2), (U=6, M=3), (U=5, M=4), (U=4, M=7), and (U=5, M=10), we will have the isoquant for Y = 105, i.e. all points on this curve will have combinations of U and M that give an output of 105 units.

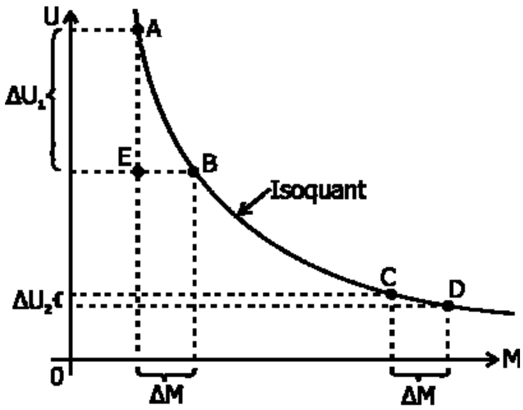

Figure 3.1 Properties of an Isoquant

Properties of Isoquants

a.) No two isoquants can intersect. This is because each isoquant represents a particular level of output, and as it is a logical contradiction to have different levels which are equal;

b.) Isoquants have negative slopes, i.e. they slope downwards from left to right. This is because, in order to remain on the same isoquant, an increase in the quantity of one input will result in a decrease in the quantity of the other input. Each input has a positive MPP; if both inputs are increased, output Y would increase and we would move to a higher isoquant.

c.) Isoquants are convex to the origin: Inputs must replace each other for output to remain at the level. As such the isoquant is convex to the origin because the rate at which one input substitutes the other becomes lower as the substitute input is used in larger quantities. In Figure 3.1, point A shows that we are using lower levels of input M. At these lower levels of the input, it can be seen that ΔM replaces a big ΔU_1 in moving from A to B. But as more of input M is used, it becomes less capable of replacing input U such that the same ΔM can now only replace a very small ΔU_2 in moving from C to D.

d.) Isoquants lying further from the origin correspond to larger amounts of output. A movement from a lower to a higher isoquant indicates an increase in output.

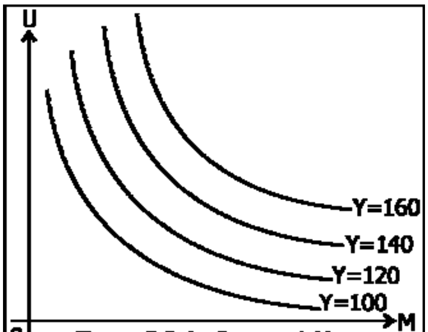

Figure 3.2 An Isoquant Map

An isoquant map is a set of isoquant curves showing technically efficient combinations of inputs that produce different levels of output. Figure 3.2 shows an isoquant map. Each isoquant on the map represents a specific level of output.

Marginal Rate of Technical Substitution

It has already been established that inputs will replace each resulting in different combinations that produce the same level of output. In our example from Table 3.1, we have five combinations of U and M to produce 105 units of output. Marginal rate of technical substitution (MRS) measures the rate at which one variable factor replaces the other leaving the level of output unchanged. Thus the

marginal rate of technical substitution <u>of manure for urea</u> (MRS_{MU}) is the number of units of urea (U) that can be replaced by one extra unit of manure (M) with output remaining unchanged.

$MRS_{MU} = \Delta U/\Delta M$.

For Y=105, the first two of combinations are (U=9, M=2) and (U=6, M=3). MRS_{MU} in moving from (U=9, M=2) to (U=6, M=3) is: $\Delta U/\Delta M = (6-9)/(3-2) = -3/1 = -3$. The answer has a negative sign because changes are in opposite directions, i.e. increasing the amounts of one input will result in a decrease in the quantity of the other input. The value or number obtained means that increasing input M by one more unit will require reducing U by 3 units in order to keep the output at 105.

Note that MRS_{MU} between these two points is the same as the <u>slope</u> of the isoquant between the points.

<u>MRS and MPP</u>: There is a relationship between MRS and MPP. With reference to Figure 3.1, it can be observed that moving from A to B can be broken into two: first, the movement from A to E, and second, the movement from E to B. From A to E, ΔU units of U are lost. Each unit's contribution to output is equal to that unit's MPP. As such, the units of output lost in moving from A to E is given as $\Delta U * MPP_U$. The second movement from E to B shows that ΔM units of M are added. With each unit of M contributing MPP_M to the output, $\Delta M * MPP_M$ units of output will be added in moving from E to B. Overall, these movements do not affect output: moving from A to E, and then E to B still leaves us on the same isoquant. Thus the output lost by moving from A to E is equal to the quantity gained in moving from E to B, i.e. $\Delta U * MPP_U = \Delta M * MPP_M$. This equation can be rearranged to become:

$\Delta U/\Delta M = MPP_M/MPP_U$. Earlier on we presented MRS_{MU} as being $\Delta U/\Delta M$.

Thus $MRS_{MU} = \Delta U/\Delta M = MPP_M/MPP_U$.

MRS measures the slope of the isoquant, as the MPPs of the inputs can be used to measure the slope of an isoquant for any given level of the inputs. From out production function:

$Y=18U-U^2+14M-M^2$, $MPP_U = \delta Y/\delta U = 18 - 2U$, while the $MPP_M=\delta Y/\delta M=14-2M$. At the Y=105 combination of (U=6, M=3), $MRS_{MU} = MPP_M/MPP_U = -[14 - 2(3)]/[18 - 2(6)] = -8/6 = -1.33$ (meaning that 1.33 units of U must be replaced by addition of one more unit of M in order to leave output (Y) unchanged at 105).

<u>TYPES OF RATE OF SUBSTITUTION</u>

a.) <u>Decreasing rate of substitution</u>: This occurs when the input being increased successively replaces smaller amounts of the other. If M is replacing U, there is decreasing rate of substitution if the first unit of M replaces more units of U than the second unit, and so on.

Decreasing rates of substitution are caused by the law of diminishing returns. In this case, the MPP of each input decreases as more units of an input as used.

In decreasing rates of substitution, the isoquant is convex to the origin as shown in Figure 3.3.

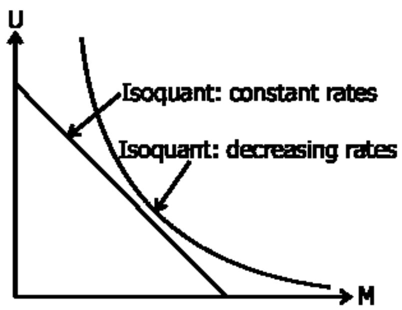

Figure: 3.3 Rates of Input substitution

b.) Constant rate of substitution: This occurs when the input being increased replaces the same amount of the other. If M is replacing U, each additional unit of M will replace the same amount of U. In this case, the isoquant is a straight line.

3.2. ISOCOST LINE

When dealing with the isoquant, concern is on the technical efficiency. Technical efficiency refers to the use of as fewer inputs as possible to produce a given output. However, firms make their decisions based on their costs and returns. As such the information about isoquants must be related to prices.

Each combination of inputs has a cost associated with it. This is the variable cost. In the case of two variable inputs, the total budget or total variable cost (TVC) = $P_UU + P_MM$. Given that $P_U = \$5$, $P_M = \$3$, TVC for the combination (U=4, M=7) is: TVC = $P_UU + P_MM = 5(4) + 3(7) = 20 + 21 = \$41$.

The price of inputs brings in the issue of economic efficiency. This is the ability to produce a given level of output at the lowest possible cost. Since it is possible to know the cost of each combination of the variable inputs, and also because there are many possible combinations of these inputs, it is possible to find those combinations that cost the same.

An isocost line is a line that represents alternative combinations of two inputs that cost the same amount. All points on the isocost line represent the various combinations of inputs. Every combination can be purchased from the same budget. In Figure 3.4, the isoquant is the line from A to G. Even though this line has an infinite number of points, we have selected points A, B, C, D, E, F, and G, each representing a specific combination of the urea (U) and manure (M). For example, point A has U_6 of input U and 0 units of input M, while point D has U_3 units of U and M_3 units of M. Because these points are on the same isocost line, the cost of each combination is the same as that of any other combination, i.e. combination A ($U=U_6$, M=0) costs the same as combination E ($U=U_2$, $M=M_4$).

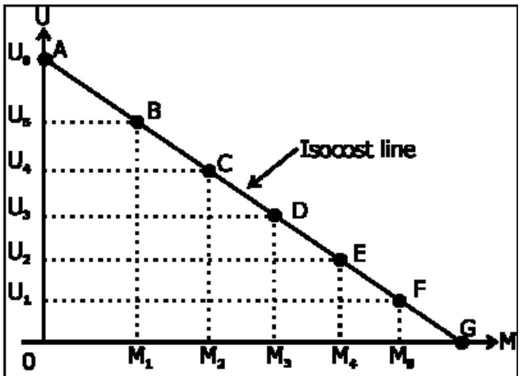

Figure: 3.4 Isocost line

Given that TVC = $P_UU + P_MM$; we can express U as the subject of the formula to have:

$$U = \frac{TVC}{P_U} - \frac{P_M}{P_U}M$$

[We have used U as the subject of the formula because it is plotted on the Y-axis.] This equation can be likened to the equation of a straight line, **y = mx + c.** U being **y**, TVC/PU being the

constant term **c**, M being **x**, and - P_M/P_U being the slope **m**. Note that the slope in the equation of a straight line is **m** the coefficient of **x**, just as - P_M/P_U the coefficient of M is the slope.
To draw the isocost line, you must know the unit cost of each input, as well as the TVC of the firm.

Example: Given that the unit price of urea, P_U = \$5, that of manure, P_M = \$3, and that the amount that the firm is prepared to spend on these inputs is \$60, the isocost line can be drawn using the following steps.
First, determine how many units of one input can be purchased when the whole TVC is spent on that input. For urea, this will be TVC/P_U = \$60/\$5 = 12; for manure, this will be TVC/P_M = \$60/\$3 = 20. This gives the intercepts, i.e. the points at which the isocost line will cut the U and M axes.
Second, the determined intercepts (U=12, M=0) and (U=0, M=20) can then be plotted. Connecting these intercepts with a straight line produces the isocost line whose slope is equal to $-P_M/P_U$ = -3/5 or -0.6 as shown in Figure 3.5 below.

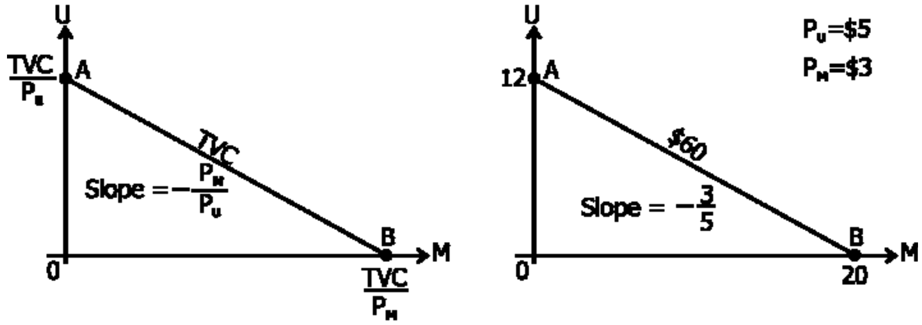

Figure: 3.5 Plotting an Isocost line

CHARACTERISTICS OF THE ISOCOST LINE

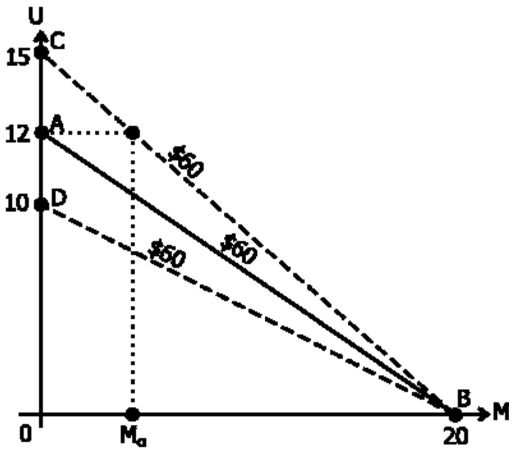

Figure 3.6 Changes in input prices

i.) The distance of the isocost line from the origin: When prices of the inputs remain unchanged, the isocost line of a bigger TVC will be far away from the origin compared to that of a smaller TVC.

ii.) When the price of one of the inputs changes (other things being unchanged), the end-point (intercept) of the isocost line will shift on the axis of the input whose price has changed. For example, when P_U reduces from \$5 to \$4, the end-point of the isoquant shifts from 12 to 15 on the axis of input U making CB the new isocost line. This is because such a reduction in price makes input U cheaper and so more of it can be purchased. Even if the firm still wants to buy only 12 units U, it will also be able to buy M_0 units of M as shown in Figure 3.6. Before the price

reduction, buying 12 of U meant that the firm bought nothing of M (as shown by point A on the AB isocost line).

If P_U is increased from \$5 to \$6, the end-point of the isoquant will shift from A to D on the axis of input U making DB the new isocost line. An increase in PU means that U becomes expensive and as such less of the input can now be purchased.

From the illustration in Figure 3.6, it can be noted that changes in input prices change the slope of the isocost line.

3.3. LEAST COST COMBINATION

You may still remember that in a factor-factor relationship a certain amount of output can be produced using different combinations of inputs. These combinations of the variable inputs (that give us the same level of output) are graphically represented as an isoquant. Even though these combinations generate the same quantity of output, they do not necessarily cost the same – each combination has its own cost. The task we have is to find the cheapest combination. By so doing we have determined the economically efficient combination of the variable inputs. Identifying the cheapest or the least cost combination can be done in two ways:

A. GEOMETRIC DETERMINATION

One way of economizing is by increasing output from a given total of resources. This is achieved when inputs are used to a point where a kwacha spent on input U contributes as much as a Kwacha spent on input M, i.e. when the marginal product per kwacha is equal for both inputs. This condition can be expressed as: $\frac{MPP_U}{P_U} = \frac{MPP_M}{P_M}$

Rearranging this equation by cross multiplication gives: $MPP_U*P_M = MPP_M*P_U$

Dividing both sides by P_U and then by MPP_U results in the equation:

$$\frac{P_M}{P_U} = \frac{MPP_M}{MPP_U}$$

In Section 3.1 (on page 21), we determined that the slope of the isoquant was MPP_M/MPP_U. In section 3.2 (on page 23), we further determined that P_M/P_U was the slope of the isocost line. As such the least cost combination occurs when the slopes of the isoquant and the isocost line are equal. This is at the point where the isocost line is tangent to the isoquant as shown by point A in Figure 3.7. As such point A (U=B, M=C) is the cheapest combination that can produce the output of 50 units as given by the isoquant.

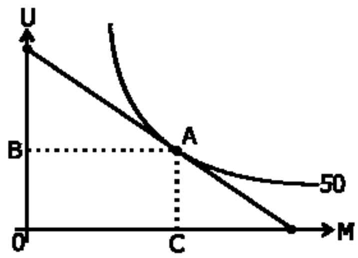

Figure 3.7 Least cost combination

Given an isoquant and the factor prices, it is therefore possible to graphically determine the point of least cost combination as well as the minimum TVC. The solution can be found by finding the point where a straight line, whose slope is the input price ratio, will be tangent to the isoquant. The point of tangency gives the least cost combination. Any one of the intercepts of such a line can be used to determine the TVC.

B. ALGEBRAIC DETERMINATION

This method also compares or equates the slopes of the isoquant to that of the isocost line. Given that $Y = 18U - U^2 + 14M - M^2$,

$MPP_U = \delta Y/\delta U = 18 - 2U$ $MPP_M = \delta Y/\delta M = 14 - 2M$

$MRS_{MU} = MPP_M/MPP_U$ is the slope of the isoquant. The price ratio P_M/P_U is the slope of the isoquant. The least cost combination is given by:

$$\frac{P_M}{P_U} = \frac{MPP_M}{MPP_U}$$

THE LAGRANGIAN METHOD

In the above methods, focus is on finding how much of each input to use in order to produce a certain amount of output given the prices of the inputs. The above methods do not consider the firm's budget.

The Lagrangian method takes into the budget limits of the firm. The method (named after Joseph Louis Lagrange who devised it) is used to find the best (or least cost) combination of two variable inputs given that the firm only has a certain amount of money to spend on the inputs.

Below is an example on how to use the Lagrangian method:

A firm's output depends on the production function $\mathbf{Y = 20H + 65F - 0.5H^2 - 0.5F^2}$ and that it only has a budget of K2,200. Determine the least cost combination of the variable inputs given that the prices of inputs H and F are K20 and K50 respectively.

Solution:

i.) Generate the constraint condition: this is an equation that gives an indication of how much of each input a firm can afford. Given that a firm only has K2,200 to spend,
 $2{,}200 = 20H + 50F$, or that $\mathbf{2{,}200 - 20H - 50F = 0}$
 20H is the amount of money that will be spent on input H, while **50F** will be spent on input F, so when we deduct these amounts from K2,200 nothing remains.

ii.) Form a new function (or equation) called Z that will combine both the production function as well as the constraint condition:
 $Z = \mathbf{20H + 65F - 0.5H^2 - 0.5F^2 + \lambda(2200 - 20H - 50F)}$

Note that the new function has introduced a new variable λ (a small Greek letter known as lamda). This variable λ is what we call the Lagrangian multiplier. This multiplier tells us the amount by which the optimum output would change when the budget increases by K1.

iii.) Compute three (3) first derivatives of the Z equation with respect to the variables H, F, λ. This means that we will have an equation for $\partial Z/\partial H$, $\partial Z/\partial F$, and for $\partial Z/\partial \lambda$. Each of these derivatives will be equated to zero as a condition to show that the Z function is at its maximum.

$\partial Z/\partial H = 20 - H - 20\lambda = 0$ .... (a)

$\partial Z/\partial F = 65 - F - 50\lambda = 0$ .... (b)

$\partial Z/\partial \lambda = 2200 - 20H - 50F = 0$ .... (c)

iv.) Compute the value of λ. This starts by making H the subject of the formula in equation (a) above such that from $20 - H - 20\lambda = 0$, **$H = 20 - 20\lambda$** .... (d)

Do the same for F from equation (b) so that **$F = 65 - 50\lambda$** .... (e)

To solve for λ, use equation (c); replace H and F in equation (c) by their respective expressions as found in equations (d) and (e):

$2200 - 20(20 - 20\lambda) - 50(65 - 50\lambda) = 0$

This is then simplified as:

$2200 - 400 + 400\lambda - 3250 + 2500\lambda = 0$

$2900\lambda = 1450$

$\lambda = 0.5$

v.) Finally to determine the least cost combination of H and F, we replace the 0.5 (the calculated value of λ) in equations (d) and (e):

H $= 20 - 20(0.5) = 20 - 10 =$ **10**

F $= 65 - 50(0.5) = 65 - 25 =$ **40**

The optimal output that can be produced is:

$Y = 20(10) + 65(40) - 0.5(10^2) - 0.5(40^2) = 1950$

The $\lambda = 0.5$ indicates that the optimal output of Y = 1950 would increase by 0.5 (to 1950.5) when the budget increases by K1 (from K2,200 to K2,201).

3.4. ISOCLINES, EXPANSION PATH, AND RIDGE LINES

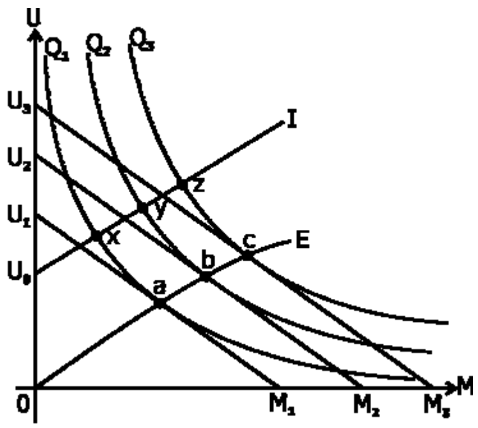

Figure 3.8 Isoquant & Isocost line map

As already indicated, an isoquant is a curve that shows all combinations of variable inputs that are capable of producing a given quantity of output. Figure 3.8 shows three isoquants, each representing a specific level of output. Isoquant Q_1 includes all those combinations of inputs U and M that can produce Q_1 units of output. The slope at each point on the isoquant measures the MRS between the inputs at that particular point.

An **isocline** is a curve or line that passes through points of equal MRS on an isoquant map. This line will connect a point on one isoquant to points on all other isoquants so long as these points have the same slope. In Figure 3.8 the line U_0I is an isocline, it cuts the three isoquants at points **x**, **y**, and **z**. In other words, the slope of isoquant Q_1 at **x** is equal to the slope of isoquant Q_2 at **y** and equal to the slope of isoquant Q_3 at **z**. As such the points **x**, **y**, and **z** are of equal MRS. An isocline therefore joins points of equal slope on an isoquant map. Because an isoquant has in infinite number of slopes (remember it is a curve), there can be as many isoclines on an isoquant map.

The **expansion path** is a special isocline. It is special in that it connects or joins the points of least cost combination for all isoquants. Points **a**, **b**, and **c** in Figure 3.8 are points of least cost combinations of U and M. The isocost line U_1M_1 is tangent to the isoquant Q_1 at **a**, U_2M_2 is tangent to Q_2 at **b**, and U_3M_3 is tangent to Q_3 at **c**. The line 0E, that passes through the points of least cost **a**, **b**, and **c**, is the expansion path. The isocost lines U_1M_1, U_2M_2, and U_3M_3 are parallel to each other. This means that the price ratio of inputs is the same for all isocost lines, i.e. the prices of inputs are unchanged. As such the slope or MRS at **a**, **b**, and **c** are the same – and this make 0E an isocline. Thus the expansion path is not just an isocline but also represents points of least cost on an isoquant map.

Features of an Expansion path

The following are features or characteristics of an expansion path:

i.) For any given price ratio of inputs, the expansion path shows how a firm's optimal input combinations change as output expands;

ii.) The expansion path can be used to distinguish between normal and inferior inputs. An inferior input is one whose quantity falls as output expands, whereas a normal input is one whose quantity increases with output;

iii.) At constant returns to scale (i.e. constant MRS), the expansion path is a straight line through the origin; and

iv.) The expansion path is based on a specific input price ratio. When input prices change by the same percentage, their ratio and the expansion path remain unchanged. The change in the price of only one input will cause both the isocost line and the expansion path to shift. For example, when the price of an input rises, each cost line rotates inward (towards the origin) on the axis of that input, i.e. the intercept shifts towards the origin. As a result, the expansion path rotates away from the axis of the input whose prise has increased. This is because a rise in price makes the input unattractive causing the firm to use less of such as input at each level of output.

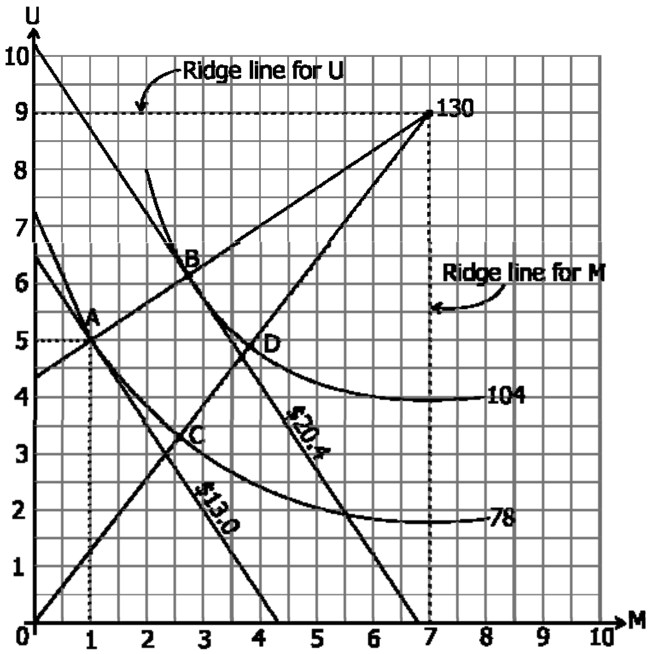

Figure 3.9 Isoclines, Expansion paths, and Ridge lines

Ridge lines are also another type of isoclines; they set limits or boundaries beyond which it becomes uneconomical to use an input. On an isoquant, inputs substitute each other at all points in order to leave output unchanged. However, it is not possible to go on and on replacing one input with another. The ridge lines are as such the boundaries to effective input substitution. In Figure 3.9, U=9 is the ridge line for input U, whereas M=7 is the ridge line for input M. These are the points where the isoquants become parallel to the axis of each input for the first time (or where MPP of each input is zero).

The line passing through points A and B in Figure 3.9 is the expansion path when P_U=\$2 and P_M=\$3. The line passing through points C and D will be the expansion path when P_U=\$9 and P_M=\$7. Maximum output is at 130 where the ridge lines and all other isoclines converge or meet.

3.5. PROFIT MAXIMIZATION IN FACTOR-FACTOR RELATIONSHIPS

Having looked at how to determine the least cost combination, you will now learn how to determine the profit maximising combination. You will recall that the expansion path marks out the least cost combinations of inputs for every possible level of output. Stated in other words, the expansion path joins up all points (that represent combinations of two variable inputs) at which it is cheapest to use the inputs as output changes. However, only one of these combinations represents the profit maximizing output. The profit maximizing conditions that we learned about in Unit 3 (from page 14 to 16) will still be used even in this case. These conditions are $P_yMPP=P_x$ for the input and MC=MR for the output.

For a production function with one output and two variable inputs, the profit equation is:
$\Pi = P_yY - P_UU - P_MM - TFC$ where $Y = f(U,M)$
At maximum profit, the slope of the profit equation is zero. This is given the first derivative of the profit equation with respect to each input:

$$\delta\Pi/\delta U = P_Y(\delta Y/\delta U) - P_U = 0, \text{ i.e. } VMP_U = P_U; \qquad \delta Y/\delta U=MPP_U$$
$$\delta\Pi/\delta M = P_Y(\delta Y/\delta M) - P_M = 0, \text{ i.e. } VMP_M = P_M, \qquad \text{recall that } P_YMPP_M = VMP_M$$

As such, profit is maximized when $VMP = P_X$ for all variable inputs used in a production process.

For example, given the production function $Y = 18U - U^2 + 14M - M^2$, and that P_Y=K4, P_U=K8, while P_M=K6. Profit is maximized when: $VMP_U = P_U$; $4(18-2U) = 8$; $U = [(8/4) - 18]/-2 = 8$
$VMP_M = P_M$; $4(14-2M) = 6$; $M = [(6/4) - 14]/-2 = 6.25$
This gives the profit maximizing level of output as $Y = 18(8) - 8^2 + 14(6.25) - 6.25^2 = 128.4$

VMP (which is P_YMPP) is the revenue that is contributed by an additional unit of input. Remember that MPP is the amount by which total output (Y) increases when one more unit of the input is used. Thus P_YMPP is the money value that an additional unit of input will contribute to total revenue (TVP). Thus by stating that $VMP = P_X$, it means that profit is maximized when all variable inputs are used to a level where each input earns (in the marginal sense) as much as it costs.

Another method of determining the optimum output is by use of cost curves. Inputs may be measured in different units e.g. land in hectares, labour in hours, herbicides in litres. Converting them in money terms standardizes the information thereby making it easy to use. Once inputs are in money terms, optimum output can be found by equating marginal cost (MC) to marginal revenue (MR or P_Y). Table 3.2 shows the various combinations of inputs U and M, as well as the output for each combination. It also includes information on total variable, average variable, and marginal costs. The information on

Table 3.2 Computing costs for least cost combination of inputs (PU=K9, PM=K7, Py=K0.65)

Least cost combinations of U and M					
U	M	Y	TVC	AVC	MC
0.00	0.00	0	0		
					0.53
0.96	0.75	26	K13.89	K0.53	
					0.58
2.00	1.55	52	28.85	0.56	
					0.72
3.30	2.55	78	47.55	0.61	
					0.95
5.00	3.90	104	72.30	0.70	
					2.22
9.00	7.00	130	130.00		

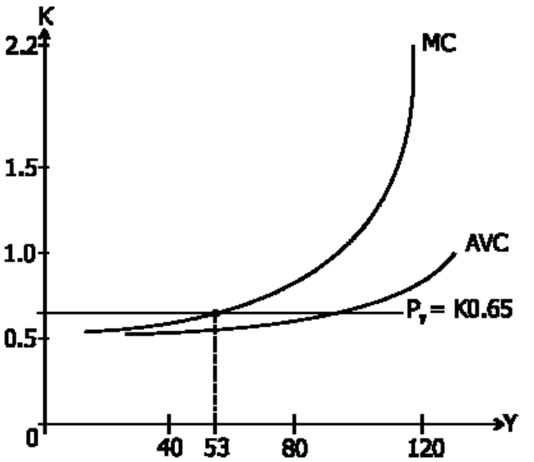

Figure 3.10 Determining the optimum of output

costs has been plotted in Figure 3.10 showing the profit maximizing output of 53 occurring where P_Y or MR is equal to MC.

MATHEMATICAL EXAMPLE: MAXIMIZING OUTPUT

Given that $Y = 18A - A^2 + 14B - B^2 + AB$

The term "AB" in the above production function is referred to as **input interaction**. Consider the MPP for each input: $MPP_A = 18 - 2A + B$;

$MPP_B = 14 - 2B + A$

Notice that the MPP of each input depends on both inputs, i.e. each MPP equation has A and B variables. This means that one input affects the MPP of the other. This results in three types of inputs or relationships between inputs. First, inputs can be <u>complementary</u>, i.e. increase in one input increases the MPP of the other – this is the case in our example. Since $MPP_A = 18 - 2A + B$, you will notice that MPP_A will increase when units of B are increased (this is because the equation for MPP_A includes the "+ B" term). Second, inputs can be <u>independent</u>. This is the case when an input does not affect the MPP of the other input. In such a case the production function does not have the input interaction (as was the case in our earlier example of $Y = 18U - U^2 + 14M - U^2$).

Inputs can be <u>competitive</u>, i.e. increase in one input decreases the MPP of the other. This is the case when the input interaction is negative.

To get back to our example, the condition for maximum output still remains as earlier presented: each input maximises output when the MPP for that input is zero. $MPP_A = 18 - 2A + B = 0$, and $MPP_B = 14 - 2B + A = 0$. Because of the input interaction it is now not possible to solve for each input separately. These equations must be solved simultaneously. The equations have been rearranged so that $18 - 2A + B = 0$ is now written as $-2A + B = -18$, while $14 - 2B + A = 0$ becomes $A - 2B = -14$:

$$
\begin{array}{r}
(-2A + B = -18)\text{-}2 \\
(\ A - 2B = -14)\ 4 \\
\hline
4A - 2B = 36 \\
-\ 4A - 8B = -56 \\
\hline
6B = 92 \\
\hline
B = 92/6 = 15.33
\end{array}
$$

Replacing B = 15.33 in either the equation of MPP_A or that of MPP_B gives A as 16.67. Thus the combination of the inputs that maximizes output is (A=16.67, B=15.33) which gives an output of 257.33 units. Another way of solving these equations is to make B the subject of the formula in the MPP_A equation:

Since MPP_A is $18 - 2A + B = 0$, making B the subject of the formula gives $B = 2A - 18$. Now instead of having B in equation of MPP_B, we will replace it with its equivalent i.e. $2A - 18$ can then be used to represent B in the equation for MPP_B which becomes: $14 - 2(2A-18) + A = 0$. This simplifies to $14 - 4A + 36 + A = 0$; which when resolved gives A as 16.67.

- Expansion path or isocline equation: As earlier indicated, an isocline joins different isoquants at points where the isoquants have the same slope or marginal rates of technical substitution (MRS). The expansion path is a type of isocline because it joints different isoquants at points of least cost. You will recall that the least cost combination is that combination at which $P_{X1}/P_{X2} = MPP_{X1}/MPP_{X2}$ (as discussed in section 3.3). Using our above (page 28) example, $MRS_{BA} = MPP_B/MPP_A$. At the least cost combination, $MRS_{BA} = P_B/P_A$. If we let P_B/P_A be equal to **r**, the point of least cost is:

  $$\frac{MPP_B}{MPP_A} = r \quad \text{Replacing the MPPs by their actual equations gives } \frac{14 - 2B + A}{18 - 2A + B} = r$$

  Taking A as the input to be plotted on the y-axis, making it the subject of the formula gives the expansion path equation: $$A = \frac{(18r - 14) + (2 + r)B}{2r + 1}$$

- Ridge line equations: Ridge lines form the boundary outside which input substitution is not possible. For each input, this boundary is where MPP is zero. Thus equating the MPP of an input to zero and expressing that input as the subject of the formula gives the ridge line equation for that input. For example $MPP_A = 18 - 2A + B$, equating this equation to zero gives $18 - 2A + B = 0$. Since this is the MPP for input A, making A the subject of the formula gives $A = 9 + ½B$ as the ridge line for input A. The ridge line equation for input B is found to be $B = 7 + ½A$.

- Profit Maximization: This requires that the value of MPP of an input equals the price of that input, i.e. $P_YMPP = P_X$. This condition must hold simultaneously for all inputs. For example, if P_Y = K0.65, P_A = K9, P_B = K7, the equations for profit maximization would be:
  K0.65(18 – 2A + B) = K9 and K0.65(14 – 2B + A). Solving these equations simultaneously (as was done for maximum level of output) gives A = 3.85, B = 3.54, and Y = 105.3

SELF HELP QUESTION 3 (15 minutes)

Given the following production function: $Y = 9A - A^2 + 6B - B^2 - AB$

- Find the combination of A and B that maximizes output;
- Given that P_Y = K3, P_A = K6, and P_B = 3, find the combination of A and B that maximizes profit.
- Derive the expansion path equation in terms of A for the given input prices

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UNIT SUMMARY

Factor-factor relationships deal with decisions on how much of several variable inputs must be used in order to maximize output or profit. The criterion for maximizing output is when the MPP for each input is zero. To maximize profit, P_YMPP must be equal to P_X for each input. Computing the level of each input that maximizes output or profit will involve solving simultaneous equations only when the production function has the input interaction.

For two or more variable inputs, the region of economic relevance still remains stage II. As such each input must only be used up to the point where its MPP is zero. This point is given by the ridge line for each input.

For a given input price ratio i.e. when given the prices of the inputs, the expansion path (equation) can be used to predict how much we must use of each input in order to produce at the lowest cost possible as we expand or increase the level of output.

4.0 PRODUCTION FUNCTIONS FOR TWO OR MORE VARIABLE OUTPUTS

Unit Introduction

In this Unit you are going to learn about the product-product relationships. This involves decisions on allocating resources in a manner that maximises revenue. In Basic Economic Theory you learned about the production possibility model. Your knowledge of the production possibility frontier will now be applied to decision making faced by firms.

Unit Learning Outcomes

When you have worked through this Unit, you should be able to:
- Distinguish the different relationships between products.
- Explain the concept of product substitution.
- Algebraically determine the revenue maximizing combination of outputs.

You have so far learned about the factor-product relationship as well as the factor-factor relationship in which the main goal was to determine the level of the variable input that must be used in order to maximise output (this being concerned with the technical efficiency of the inputs) or to maximize profit (this being an issue of economic efficiency). Under the product-product relationship, you will learn about decisions on how to allocate a given quantity of resources among alternative uses. Product-product relationships involve the production of two or more variable output from a given set of resources. It is about deciding what to produce and therefore how to allocate resources among the different outputs. As such you will learn about how a given batch of variable and fixed inputs can be shared among the enterprises so as to produce optimally. This is different from input-output, or factor-factor relationships that deal with how inputs should be allocated with within an enterprise.

Much as a firm can have more than two enterprises, we will consider the case of two products. The production possibility curve (PPC) is used to depict two production functions on one graph. The PPC is also called the Transformation Curve. Thus if there are two products Y_1 and Y_2, the manager has to determine how much inputs to used on each product. The primary use of the PPC is to determine the most profitable combination of enterprises given limited amounts of inputs.

DERIVING A PPC FROM PRODUCTION FUNCTIONS

A PPC shows the combinations of products that can be produced using a given amount of inputs. For this reason, the PPC is also called an isoresource curve because all points on the curve represent combinations outputs that can be produced using equal (iso-) amounts of inputs. These points on the PPC also represent the maximum amount of output.

Table 4.1 Production Possibilities

Column 1			Column 2			Column 3	
						Possibilities for X = 7	
X	Y_1	MPP_{XY1}	X	Y_2	MPP_{XY2}	Y_1	Y_2
0	0		0	0		0	43
1	7	7	1	12	12	7	42
2	13	6	2	22	10	13	40
3	18	5	3	30	8	18	36
4	22	4	4	36	6	22	30
5	25	3	5	40	4	25	22
6	27	2	6	42	2	27	12
7	28	1	7	43	1	28	0
8	27	-1	8	42	-1		

Table 4.1 shows the different amounts of Y_1 and Y_2 that can be produced when different levels of input X are used. Column 1 shows units of Y_1 produced at different levels of X, as well as the marginal product of X in the production of Y_1 (i.e. MPP_{XY1}). Column 2 shows the same details for Y_2. In other words, each of the first two columns represents a separate production function.

Thus if we had 7 units of X, using all of them on Y_1 gives an output of 28. The same 7 units of X if used only on Y_2 will produce 43 units of output. This is shown by the first and last lines of Column 3. Column 3 shows how the 7 units of X can be used to produce different combinations of Y_1 and Y_2. For example the output combination (Y_1=18, Y_2=36) i.e. the fourth line of Column 3, indicates that 18 units of Y_1 will be produced from 3 units of X, with the other 4 units of X (out of 7) producing the 36 units of Y_2.

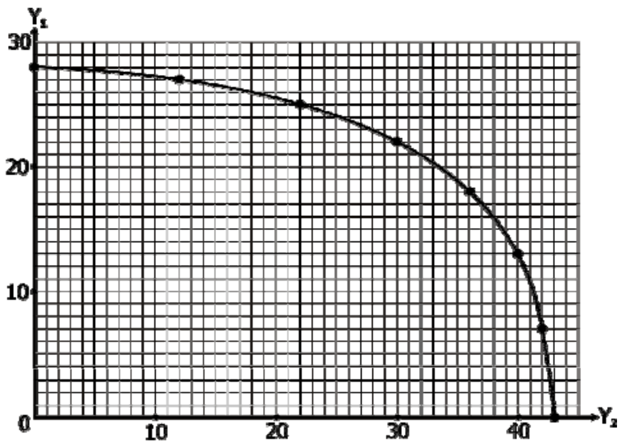
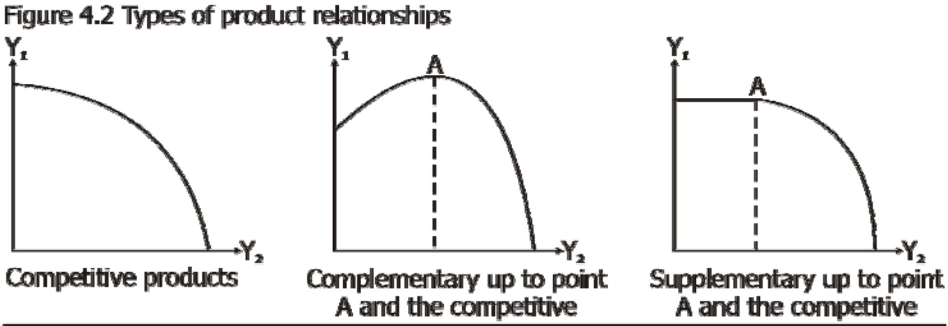


Figure 4.1 PPC based on data from Column 3 of Table 4.1

In fact, Column 3 is the production possible schedule for 7 units of X, it shows the different combinations of Y_1 and Y_2 that can be produced from 7 units of the input X. Figure 4.1 shows the PPC drawn using information from the schedule in Column 3.

4.1 RELATIONSHIPS AMONG PRODUCTS

Figure 4.2 Types of product relationships



A production possibility curve (PPC) can show the types of relationships that exist among products. There are three such relationships:

- Competitive products: these are products that compete for a resource. Thus increase in the production of one product results in the decrease in the production of the other product. In this case the PPC has a negative slope. A field crop enterprise will compete with beef enterprise for land - more land for the crop means less land for pasture.

- Complementary or joint products: Two products are complementary if an increase in one causes an increase in the other product, so long as the amount of inputs used is held constant. Production of beef and dairy products increases the production of hides. Similarly hay and wheat are complementary. In an intercrop or rotation of beans and maize, growing beans increases maize yield to some extent. Beyond a certain point, the two crops will become competitive.
- Supplementary products: Two products are supplementary if the amounts of one can increase without increasing or decreasing the amount of the other. This occurs when enterprises use different resources, or when they use the same resources at different times. An example could be rearing sheep and goats on a piece of land that is not suited for crops.

4.2 MARGINAL RATE OF PRODUCT SUBSTITUTION

Points along the PPC represent combinations of outputs that can be produced from a set of resources. In the case of competitive products, moving from one point to another implies that units of one product are increased while those of the other product are lost, i.e. products substitute each other. The marginal rate of product substitution (MRPS) refers to the amount by which one output changes in quantity when the other output increases by one unit along a PPC. Given outputs Y_1 and Y_2, the marginal rate of product substitution of Y_2 for Y_1 ($MRPS_{Y2Y1}$) is given by $\Delta Y_1/\Delta Y_2$. MRPS measures the slope of the PPC. MRPS is also called the marginal rate of transformation (MRT). MRT shows the rate at which one product (e.g. Y_1) can be transformed into another product (e.g. Y_2). For example, point V in Figure 4.3 has an absolute slope equal to 2. This means that the firm can increases the production of food (Y_2) by 1 unit only if it sacrifices 2 units of clothing (Y_1). Thus the MRT of clothing into food is equal to $\Delta Y_1/\Delta Y_2$. MRT of clothing into food is also known as the **opportunity cost** of food in terms of clothing.

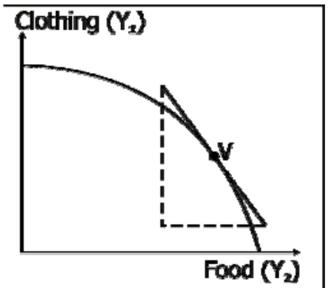


Figure 4.3 Product substitution

- The concave shape of the PPC reflects the fact that as production of food increases (and that of clothing decreases), more and more units of clothing will be given up for every extra unit of food produced (Law of increasing opportunity cost).
- MRT of clothing into food is also equal to MC_F/MC_C (where MC is the marginal cost)
- When MRPS is positive, products are complements; it is negative for competitive products; and it is zero or undefined for supplementary products.

4.3 ISOREVENUE LINE

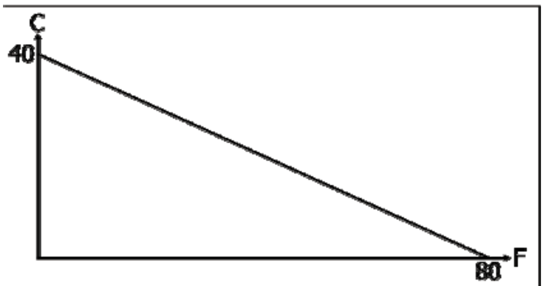


Figure 4.4 Isorevenue line: P_C=$2, P_F=$1, TR=$80

Given two products Y_1 and Y_2, and their respective prices as P_{Y1} and P_{Y2}, total revenue (TR) is given as TR = $P_{Y1}Y_1 + P_{Y2}Y_2$. Using our example of clothing (C) and food (F), and assuming that the amount of possible TR is \$80, the unit price of clothing (P_C) is \$2, and that of food (P_F) is \$1, it is possible to determine combinations of clothing and food that can generate revenue of \$80. If all the \$80 was generated only from clothing, then

$80/$2 = 40 units of clothing would have to be produced; if generated only by food, then $80/$1 = 80 units of food would have to be produced. Plotting these extreme combinations or intercepts of (C=40, F=0) and (C=0, F=80) results in an isorevenue line as shown in Figure 4.4. All points on the isorevenue line represent combinations of clothing and food that can generate total revenue of $80. Note that the intercepts can be found by TR/P_C on the clothing axis, and TR/P_F on the food axis. The slope of the isorevenue line is determined by the prices of output as $-P_F/P_C$.

4.4 THE REVENUE OR PROFIT MAXIMIZING COMBINATION OF OUTPUT

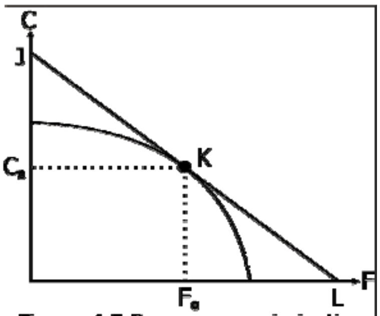


Figure 4.5 Revenue maximization

The revenue maximizing combination of outputs is at the point where the isorevenue line is tangent to the PPC as shown by point K in Figure 4.5. This gives the combination ($C=C_0$, $F=F_0$) as the revenue maximizing combination of clothing and food. Mathematically, this can be obtained by equating the slope of the PPC to that of the isorevenue line: $MRPS_{FC} = \Delta C/\Delta F$ is the slope of the PPC, while $-P_F/P_C$ is the slope of the isorevenue line. As such, maximum revenue is obtained when $\Delta C/\Delta F = P_F/P_C$.

$\Delta C/\Delta F = P_F/P_C$ can be rewritten as $P_C\,\Delta C = -P_F\,\Delta F$

Maximizing revenue also maximizes profit because all the possible combinations of output are produced from a certain fixed bundle of inputs. As such the combination that generates the highest revenue is the most profitable combination.

Table 4.2 Production possibilities and total revenue

Production Possibilities for X = 7		Revenue from Clothing (C) P_C = $2	Revenue from Food (F) P_F = $ 1	Total Revenue
Clothing	Food			
0	43	0	43	43
7	42	14	42	56
13	40	26	40	66
18	36	36	36	72
22	30	44	30	74
25	22	50	22	72
27	12	54	12	66
28	0	56	0	56

Table 4.2 shows the various combinations of clothing and food that can be produce from 7 units of some input. For each combination, total revenue (TR) has been computed, and the maximum TR as can be seen from the table is $74 when 22 units of clothing and 30 units of food are produced. Mathematically, this is obtained based on the condition that the slope of the PCC is equal to the slope of the isorevenue line, i.e. $\Delta C/\Delta F = -P_F/P_C$.

4.5 OPPORTUNITY COST AND MARGINAL CRITERION FOR RESOURCES ALLOCATION

As already indicated, revenue is maximized when $\Delta Y_1/\Delta Y_2 = P_{Y2}/P_{Y1}$, i.e. when the slope of the PPC is equal to that of the isorevenue line; P_{Y1} and P_{Y2} being the respective unit prices of products Y_1 and Y_2. This can be rewritten (after cross multiplication) as:

$$P_{Y2}\Delta Y_2 = P_{Y1}\Delta Y_1$$

A decrease in Y_1 occurs only because resources are being moved from Y_1 to the production of Y_2. The amount of the input being shifted can be denoted as ΔX. Dividing each side in the above equation by ΔX gives: $(P_{Y2}\Delta Y_2/\Delta X) = (P_{Y1}\Delta Y_1/\Delta X)$.

Notice that dividing both sides by the same value i.e. ΔX is mathematically correct. Apart from this, doing so enables to introduce a new aspect to our revenue maximising condition. You will recall that $\Delta Y/\Delta X$ is the MPP, as such our equation becomes: $P_{Y2}MPP_{XY2} = P_{Y1}MPP_{XY1}$.

Earlier on in Unit 2 (on page 15) we indicated that P_YMPP is also called the value of the marginal product (VMP). And so our equation can now be stated as: $VMP_{XY2} = VMP_{XY1}$.

VMP is the revenue generated by using one more unit of an input – it is MPP in monetary terms. This means that revenue is maximized when input use is such that an extra unit of the input used generates the same revenue regardless which enterprise it is allocated to.

In this situation, the VMP for the product being sacrificed is equal to the VMP for the product being gained. And so the opportunity cost for both enterprises must be equal for total revenue to be maximized.

<u>EXAMPLE</u>:

Given the product-product relationship as $\mathbf{Y_1 = 100 - 0.0065Y_2^2}$ and that P_{Y1} = K5 and P_{Y2} = K6, determine how much of each product can be produced without production of the other. How much of each product must be produced in order maximize revenue?

<u>Answer</u>:

When all the given inputs are used on only one product, there will be nothing of the other. Thus if we are to produce only Y_1, zero of Y_2 will be produced, as such we replace Y_2 with zero in given equation:

Y_1 = 100 when Y_2 is zero.

If the firm only produced Y_2, the amount of Y_2 produced will be: $(100/0.0065)^{½} = 124.03$

The combination of the two products that maximizes revenue is obtained as: $\Delta Y_1/\Delta Y_2 = - P_{Y2}/P_{Y1}$

$\Delta Y_1/\Delta Y_2$ is computed as the first derivative of the given equation:

$$\delta Y_1/\delta Y_2 = - 0.013Y_2; \; - P_{Y2}/P_{Y1} = - 6/5$$

Thus revenue is maximum when $- 0.013Y_2 = - 6/5$; $Y_2 = 92.3$

Y_1 can now be found by using 92.3 in place of Y_2 in the original product-product equation:

$$Y_1 = 100 - 0.0065(92.3^2) = 44.6$$

<u>SELF HELP QUESTION 4</u> (15 minutes)

Given the production possibility for Beans (B) and maize (M) as $B = 72 - 0.0005M^2$

- Determine how much maize will be produced if no beans is produced;
- What is the maximum revenue possible if the unite prices for beans and maize are \$2.5 and \$4 respectively.

<u>UNIT SUMMARY</u>

Product-product relationships involve the production of two or more variable output from a given set of resources. It is about deciding what to produce and therefore how to allocate resources among the different outputs. The objective in product-product relationships is to determine the revenue maximizing combination of enterprises. In the various analyses and computations done, focus was only on competitive products. As such given two competitive products Y_1 and Y_2, revenue (and therefore profit) is maximized when $\Delta Y_1/\Delta Y_2 = - P_{Y2}/P_{Y1}$.

5.0 UNCERTAINTY AND RISK

Unit Introduction

Throughout the module, we have assumed that prices are stable and that producers have perfect certainty. This assumes that nothing unknown can happen, that producers know the eventual outcome of their production processes even before production begins. This is however not true in real life situations; many things may not be known with certainty while others may not be known altogether. In this Unit you are therefore going to learn about uncertainty as well as risks that might be faced by firms in agriculture.

Unit Learning Outcomes

When you have worked through this Unit, you should be able to:

- Distinguish between risk and uncertainty.
- Explain the types of risks a firm can face.
- Describe the decision making process in situations of uncertainty.
- Explain the various ways of reducing risk and uncertainty

To begin with, we would like to explain the basis for risks and uncertainty. Knowledge is in two general forms namely perfect and imperfect knowledge. When there is perfect knowledge, then everything about an event (including its outcomes) is known with absolute certainty. When knowledge is imperfect, then some factors about an event are known with certainty, others can be predicted, while others are unknown altogether.

Generally, risk is related to the expected losses which can be caused by an event, and to the likelihood of this event happening. The harsher the loss, and the more likely the event, the worse the risk; i.e. risk = (the probability that some event will occur) x (the consequences if it does occur). Risk implies that some factors are known with certainty, while other factors are not known with certainty but there is a known chance or probability of their occurrence.

Uncertainty is the situation in which there is neither certainty nor a known chance concerning the occurrence of an event.

A decision is called *risky* when the probabilities that certain states will occur in the future are precisely known. For example betting in games that involves rolling of a die is risky. This is because the possible outcomes and their chances are known. In contrast, a decision is called *uncertain* when the probabilities are not precisely known e.g. the outcomes of sports events. As such, risk and uncertainty can be distinguished by the degree with which probabilities are known: in case of uncertainty, probabilities are not precisely known even though people can form more or less vague beliefs about such probabilities. The second difference is that risk always involves possibility of some loss, but uncertainty does not always involve losses, sometimes the unknown event or outcome may be good.

In a nutshell, uncertainty may involve things that are completely unknown, whereas risks are often understood in terms of calculable probabilities. It is however, common to use the terms risk and uncertainty interchangeably.

5.1 Types of Risks and Uncertainty

Risk has always been a part of agriculture. Increasingly, agriculture today is full of new rules, new stakes, and most of all, new risks.

Risks and uncertainties can be classified in a number of ways. Some authors use two categories such as business risks (made up of price and production risks) and financial risks (associated with debt financing). A widened classification gives five general types of risks as: production risk, marketing risk, financial risk, legal risk, and human resources risk.

Production (or Biological) Risk derives from the uncertain natural growth processes of crops and livestock. Weather, disease, pests, and other factors affect both the quantity and the quality of commodities produced.

Marketing (or Economic) Risk refers to uncertainty about the prices producers will receive for commodities or the process they must pay for inputs. The nature of price risk varies significantly from commodity to commodity.

Financial Risk results when the farm business borrows money and creates an obligation to repay debt. Rising interest rates, the prospect of loans being called by lenders, and restricted credit availability are also aspects of financial risk.

Legal Risk results from uncertainties surrounding government actions. Tax laws, regulations for chemical use, rules for animal waste disposal, and the level of price or income support payments are examples of government decisions that can have a major impact on the farm business. (Some authors combine financial and legal risks under the heading "Institutional Uncertainties).

Human Resources Risk refers to factors such as problems with human health or personal relationships that can affect the farm business. Accidents, illness, death, disability, and divorce are examples of personal crises that can threaten a farm business.

5.2 Risk Attitudes

A person's approach or mind-set towards risk constitutes his/her risk attitude. A decision maker's risk attitude determines his/her willingness to engage in risky prospects. Risk attitudes can be divided into three types: risk averse, risk preferring, and risk neutral.

A decision maker displays risk aversion if he is more cautious with a preference for less risky events/plans even if it means giving up some expected income. Risk referrers or takers are more adventuresome with a preference for more risky business alternatives; given alternatives that have similar requirements, they will select an alternative that has a higher outcome even with the knowledge that it is more risky. The risk neutral individual is one who is indifferent (or unconcerned) about the risk; he/she will choose the most rewarding alternative without regard for whatever risks it may have.

It is worth noting than an individual may not be in the same category for all decisions i.e. one may be risk averse in one situation but become a risk taker in another situation. Furthermore, risk attitudes change over time as farmers change from one goal to another. Risk attitudes also change depending on a farmer's financial position, e.g. a farmer may become more risk averse following a year of large losses resulting in declines in his/her net worth.

5.3 Components of a Risky Decision Problem

Decision making under uncertainty has several components. Thus to evaluate a risky problem, it should be broken into its components which are:

1. Alternative actions that a decision maker might take (e.g. having different application rates of a given fertilizer),
2. State or possible future events that might occur for each alternative which the decision maker cannot control (e.g. alternative weather and price conditions such as normal, favourable and unfavourable),
3. Payoff, outcomes, or consequences of actions taken for each alternative (such as gross margins per acre derived by deducting variable costs from the gross return),
4. Probabilities associated with each event (based on past/historical data)

With this information, is possible to know the outcomes (gross margins) under different events (such as weather and price conditions) which on the basis of the probabilities of each event, help determining the appropriate choices for the various risky conditions.

5.4 Safeguards against Risk and Uncertainty

The various measures used to counter risk and uncertainties include:

1. <u>Selecting enterprises with low variability</u>: Certain enterprises tend to have stable prices and yields. Inclusion of such enterprises is a good way of reducing risk and uncertainty.
2. <u>Insurance</u>: Farmers can now insure their premises, crops and livestock against a number of risks such as fire, theft, and other situations that individual insurers provide cover for.
3. <u>Forward contracts</u>: These ensure that prices of both products and inputs are agreed on in advance of delivery and use. This enables the farmer to take advantage of current prices especially if product prices are expected to fall in future, or input prices expected to rise.
4. <u>Flexibility</u>: this refers to the ease with which the firm can switch from product to product, or in terms of time taken to switch from one system of production to another.
5. <u>Diversification</u>: This involves dealing in a range of enterprises so that a farmer does not only depend on one. Losses in one enterprise may be countered by gains in other enterprises. It enables the farmer to harness synergies e.g. by having a fish farm and a piggery so that the pig excreta is used to fertilize fish ponds.
6. Maintenance of financial and resource reserves.

APPENDIX 1: ANSWERS TO SELF-HELP QUESTIONS

QUESTION 1:

- Boundary between stages I and II cannot be defined $X = (-37/0.001)^{1/2}$
- Boundary between stages II and III: X = 400. This is found using $\delta Y/\delta X=0$ i.e. when MPP=0
- Stage of production when X=15: MPP is 0.77, APP is 3.252. $E_p=(0.77/3.252)=0.2368$. X=15 is in stage II because E_p is between 0 and 1.

QUESTION 2:

- E_p when X=40. At X=40, MPP=1.6 and APP=3.55. E_p = MPP/APP = 1.6/3.55 = 0.45. The intepretation is that since E_p is between 0 and 1 X=40 is in stage II, OR that at X=40 increasing the input by 1% increases the output by 0.45%.
- Level of X that maximizes profit (when P_Y=100 and P_X=10) is 190. It is obtained using $P_Y MPP=P_X$

QUESTION 3:

- A and B when output is maximum: A=4, B=1. This is found using the MPP of each input and equating it to zero. The results are two equations that can be solved simultaneously.
- A and B which maximize profit (when P_Y=3, P_A=6, P_B=3): A=3, B=1. This is found $P_Y MPP=P_X$ for each input. Again the result are two equations that can be solved simultaneously.
- $A = [2(3–B) – r(9+B)]/(1–2r)$ when $r=P_B/P_A$ OR $A = [3(3–2r) – B(1–2r)]/(2–r)$ when $r=P_A/P_B$. In the first case the equation is obtained by making A the subject of the formula based on the condition that $MPP_B/MPP_A = P_B/P_A$. In the second case the equation is obtained using $MPP_A/MPP_B = P_A/P_B$.

QUESTION 4:

- Quantity of maize when beans is not produced: 120. This is obtained by replacing B with 0 and then solving the equation for M.
- Revenue maximizing level of beans is 70.72, that of maize is 16. The maximum revenue is \$240.80. This is computed using $\Delta B/\Delta M = - P_M/P_B$, where $\Delta B/\Delta M$ is found as the first derivative of the given product-product function.

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